

TRABAJO ESPECIAL DE GRADO

PROCESAMIENTO DIGITAL DE LA SEÑAL OBTENIDA A PARTIR DE UN DISPOSITIVO DETECTOR MONOFOTÓN SUPERCONDUCTIVO “TRANSITION-EDGE SENSOR”

Presentado ante la Ilustre
Universidad Central de Venezuela
por la Br. Delgado A., Maria T.
para optar al Título de
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Torino. 2008. pp. 139.

Palabras Clave: Digital Signal Processing; Single-Photon detectors; Transition-Edge
Sensors; Pulse Height Analysis; Energy Resolution.

Resumen. La detección eficiente de fotones únicos es esencial en todas las aplicaciones referidas al campo de la fotónica cuántica, tales como información y criptografía cuántica, circuitos de distribución de claves cuánticas, comunicaciones interplanetarias y muchas otras. Para esto, es necesario contar con dispositivos fotodetectores que posean una buena resolución energética intrínseca y la capacidad de discriminar el número de fotones incidentes sobre ellos. En la actualidad, los mejores detectores de fotones han sido desarrollados a partir de films superconductores de dimensiones reducidas (μm^2) que aprovechan su rápida transición de fase del estado superconductor al estado normal para detectar la incidencia de un único fotón sobre ellos. Estos dispositivos se conocen con el nombre de *Transition-Edge Sensors (TES)*. En este Trabajo Especial de Grado se presentan diversas técnicas de procesamiento digital aplicadas a la señal obtenida a partir de un dispositivo superconductor TES fotodetector construido y caracterizado en el Laboratorio “Innovative Cryogenic Devices” del *Istituto Nazionale di Ricerca Metrologica di Torino*, con el objetivo de evaluar y mejorar la resolución energética del mismo. Para esto fueron considerados los casos en los cuales la señal procesada fue generada por una fuente monocromática y por un laser modulado externamente con longitudes de onda de 200 nm y 690 nm respectivamente, funcionando a niveles de potencia muy bajos, y temperaturas menores a los 100mK.

POLITECNICO DI TORINO

III Facoltà di Ingegneria

Corso di Laurea in Ingegneria delle Telecomunicazioni
Riassunto Tesi di Laurea

**Digital Signal Processing of Data from a
Superconducting Transition Edge Sensor Single Photon Detector**



Relatore: Prof. Roberto Gaudino
Candidato: Maria Teresa Delgado
Novembre 2008

I. SUMMARY

Efficient single-photon detection is essential for any photonic quantum information processing application. High counting-rate detectors capable of sensing single photons in the infrared region are needed for several applications in different fields such as high-bandwidth interplanetary communications, test of high-speed semiconductor circuits and quantum key distribution.

Transition-edge sensors (TES) are microcalorimeters that exploit their sharp transition from the superconducting to normal state to detect single photon. They are promising for their application in the field of quantum information and quantum computation thanks to their outstanding capabilities. Good energy resolution and photon number discrimination at visible and near-infrared wavelengths make them extensively used as single photon detectors.

Different TES's consisting of Ti/Pd/Ti multilayers have been fabricated and characterized by the "Cryogenic Innovative Devices" group at the *Istituto Nazionale di Ricerca Metrologica* (INRiM). Among these a Pd/Ti TES has been characterized as single photon detectors with a transition temperature of $T_c = 105mK$.

This detector was irradiated with both pulses $0.5\mu s$ wide from a 690nm modulated diode laser, and from a 200nm monochromatic source, attenuated by proper neutral density filters, coupled to a $50\mu m$ multimodal optical fiber aligned with the detector at a distance of $600\mu m$.

The signal obtained from this detector, which constitutes the object of study of this thesis will be digitally processed in order to evaluate the performance of the TES single photon detector

Digital Signal Processing (DSP), as the term suggests, is the processing of signals by digital means. DSP is the mathematics, the algorithms, and the techniques used to manipulate the signals after they have been converted into a digital form.

The field of Digital Signal Processing has expanded significantly over the last few decades as a result of rapid developments in computer technology and integrated circuit fabrication, constituting a vast and fascinating subject which has exploded in application in recent years.

DSP includes many different topics such as: digital filters, analysis of signals and systems (especially in terms of frequency), synthesis of signals, detection of signals and estimation of

signal and system parameters, data compression and so on.

Computer-based software is essential in several areas of DSP. Software allows the user to experiment with DSP: non-real-time signal processing can be performed with the assistance of a computer using a variety of software. In addition, computer-based software can be used to generate the real-time software that will run on a dedicated DSP microprocessor.

Although some of the mathematical theory underlying DSP techniques, such as Fourier and Hilbert Transforms, digital filter design and signal compression, can be fairly complex, the numerical operations required actually to implement these techniques are very simple, consisting mainly of operations that could be done on a four-function calculator. Convolution and Discrete Fourier Transform are also among the principal operations of DSP.

In its simplest form, DSP techniques can be implemented in the digital domain, with far greater precision and stability than their analogue counterparts, and very often at much lower cost.

The main purpose of this thesis is developing and proposing a DSP computer-based routine that allows processing the photo-detector received signal with the aim of performing several operations on it and improving its quality, and thus obtaining a better energy resolution of the TES device.

The tool employed to achieve this objective consists on a series of software routines developed under Matlab® environment that includes programming codes to load the data, perform digital filtering, detect incoming pulses, and compute histograms and different data statistics.

While the first two chapters of this work can be seen as a theoretical preamble on Superconducting Transition Edge Sensors and DSP Techniques, the following chapters correspond to the description of the experimental setup and to the characterization, analysis and process of the signal from the TES photo-detector, object of study.

The stages of the DSP computer-based algorithm are developed and justified by their respective theoretical basis, the results obtained when processing the signal from the TES

photodetector with the corresponding Matlab® routines are presented and discussed. To conclude, it is established a comparison between the energy resolution of the TES detector obtained without any processing of the signal and the one obtained when applying the Digital Signal Processing (DSP) algorithm.

II. DSP ALGORITHM AND RESULTS

Data Acquisition

The data acquisition process is performed by using a LeCroy oscilloscope model WR44xi that allows storing the data in *ascii* format

Pulse Processing

The purpose of the pulse processing routine developed is analyzing the raw data from the detector and storing the useful information contained in every event, such as the pulse amplitude and the pulse width.

Pulse-detection

In the case of a non-triggered source, it is necessary to determine the occurrence of an event: the DSP software routine watches the incoming data and detects the pulses that result when a photon hits the detector. When in a data frame an event is detected, the program collects the entire pulse and stores it along with its coordinates (temporal position and data frame number) for its later analysis. A pulse height analysis is then performed in order to get the best possible estimate of the height of each pulse.

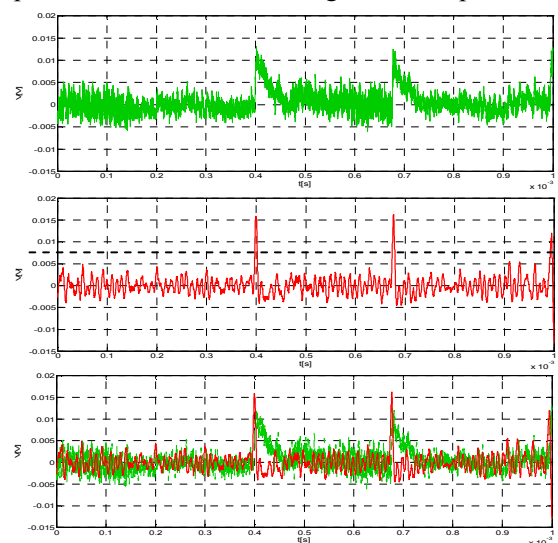


Figure 1. Pulse-detection Algorithm

Pile-up Rejection

The fact that pulses from the radiation detector are randomly spaced in a time can lead to interfering effects between pulses when counting rates are not low. These effects are called pile-up.

When looking at the pulse-detection records as the one showed before on Figure 2, it can be noticed that the pulse pile-up phenomenon is present in a significant number of them. This represents a problem when trying to obtain a good template of the pulse waveform. A pile-up rejection routine has been developed.

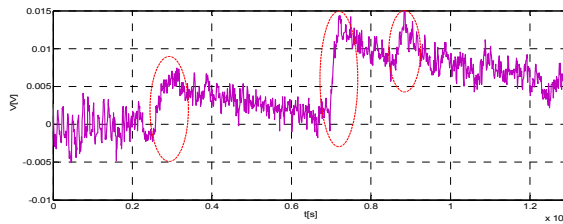


Figure 2. Pile-up phenomenon present on data frames

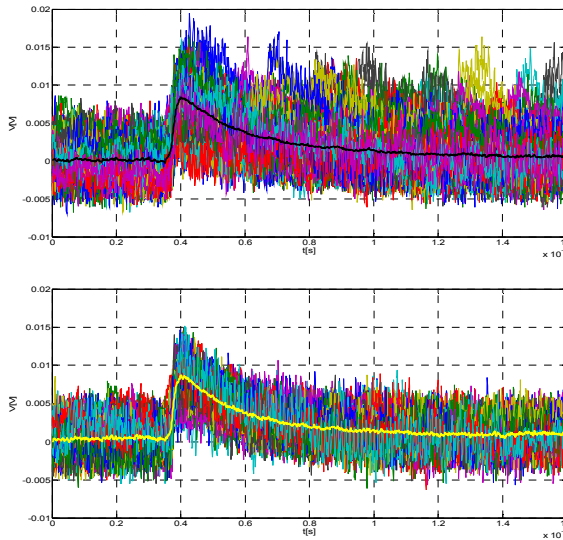


Figure 3. Pile-up rejection Algorithm; monochromatic source.

Signal Filtering

With the aim of obtaining a good template of the pulses, digital filtering is applied to the signal: low pass filtering and optimal filtering (matched filter and Wiener Filter) is performed in order to achieve a good signal quality, that later will be traduced on a high-quality pulse templates.

Digital Baseline Restoration

To make a good estimation of the amplitude of the pulses detected by the TES photodetector, it is necessary to perform a restoration of the baseline

Common baseline estimation is made for the pulses detected, from each pulse record a significant number of samples before the generated trigger were stored for this calculation.

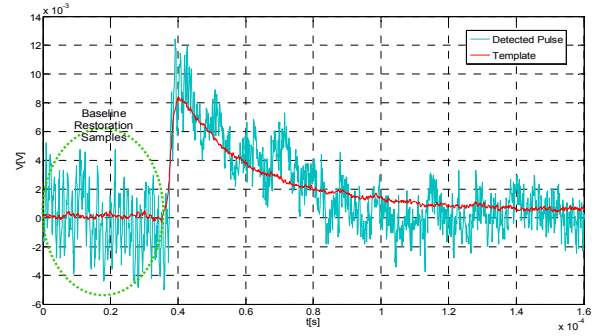


Figure 4. Baseline Restoration

Signal Templates

Once the signals have been processed, it is possible obtaining a template of the pulse shape. The template of the signal from the monochromatic source is show in Figure 3. When using the triggered source, the pulses templates obtained are shown in Figure 5.

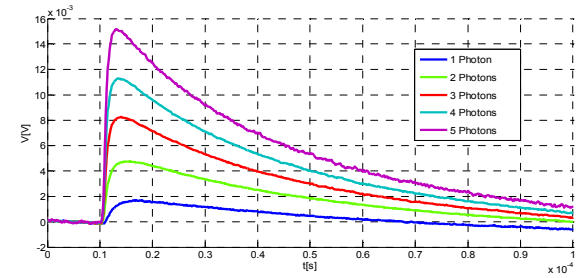


Figure 5. Pulses Templates obtained when using the triggered source

Signal Histograms

Finally, the energy resolution can be evaluated by comparing the values of the FWHM (full-width-at-half-maximum) of the Gaussians obtained when computing the pulse heights histograms. In Figure 6, the histogram obtained when using the monochromatic source is shown, while in Figure 7 and 8 the histograms obtained when working with the laser (triggered source) are presented.

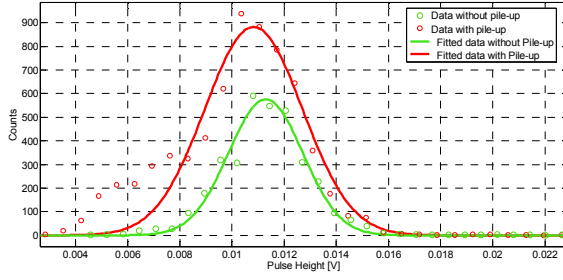


Figure 6. Sample Probability Density Function of the pulse heights when using a monochromatic source

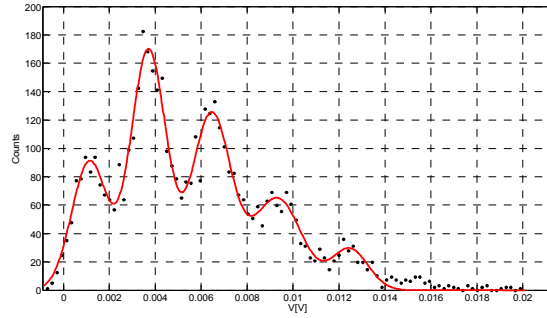


Figure 7. Histogram of the pulses heights without DSP

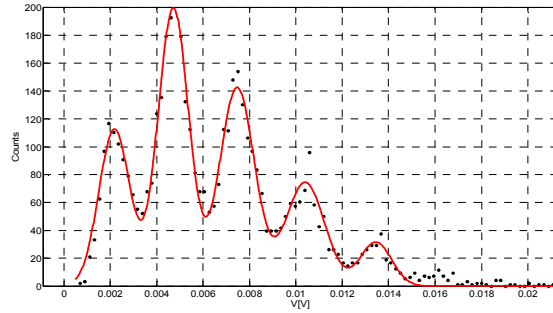


Figure 8. Histogram of the pulses heights without DSP

		Pulse Height[mV]	FWHM[mV]	Energy Resolution(%)
NO DSP	1 Photon	1.132	1.8265	161.3
	2 Photons	3.699	1.8211	49.232
	3 Photons	6.437	2.0173	31.336
	4 Photons	9.305	2.4917	26.777
	5 Photons	12.467	1.8731	15.024
DSP	1 Photon	2.175	1.5736	72.325
	2 Photons	4.717	1.5583	33.037
	3 Photons	7.463	1.7677	23.684
	4 Photons	10.400	1.9772	19.011
	5 Photons	13.445	1.6015	11.911

Table 1. FWHM of the Peaks from the histograms presented in Figures 7 and 8

III. CONCLUSIONS

The DSP algorithm proposed in this work has demonstrated to improve the energy resolution of the signal obtained from the TES single-photon detector fabricated and characterized by the Innovative Cryogenic Group at the INRiM.

When applying the DSP tool developed in this work to the data corresponding to the signal from the triggered source the FWHMs of the histogram peaks reduce their value (see Table 1); this fact translates directly to an energy resolution improvement.

It is worth mentioning that the computer-based tool developed in this thesis can be used to generate real-time software to be run on a dedicated DSP microprocessor. It could be useful implementing the DSP algorithm proposed in this thesis in a microprocessor chip.

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INTRODUCTION

Efficient single-photon detection is essential for any photonic quantum information processing application. High counting-rate detectors capable of sensing single photons in the infrared region are needed for several applications in different fields such as high-bandwidth interplanetary communications, test of high-speed semiconductor circuits and quantum key distribution.

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Different TES's consisting of Ti/Pd/Ti multilayers have been fabricated and characterized by the "Cryogenic Innovative Devices" group at the *Istituto Nazionale di Ricerca Metrologica* (INRiM). Among these a Ti/Pd/Ti TES with a transition temperature of $T_c = 105mK$ has been characterized as single photon detector.

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The main purpose of this thesis is developing and proposing a DSP software routine that allows processing the photo-detector received signal with the aim of performing several operations on it and improving its quality, and thus obtaining a better energy resolution of the TESs.

Software routines are essential in several areas of DSP; they allow the user to experiment with DSP: non-real-time signal processing can be performed with simply the assistance of a computer using a variety of software. In addition, computer-based software can be used to generate the real-time software that will run on a dedicated DSP microprocessor.

Although some of the mathematical theory underlying DSP techniques, such as Fourier and Hilbert Transforms, digital filter design and signal compression, can be fairly complex, the numerical operations required actually to implement these

techniques are very simple. Convolution and Discrete Fourier Transform are also among the principal operations of DSP.

In its simplest form, DSP techniques can be implemented in the digital domain, with far greater precision and stability than their analogue counterparts, and very often at much lower cost.

The tool employed to achieve this objective consists of a series of software routines developed under Matlab® environment that includes programming codes to load the data, perform digital filtering, detect incoming pulses, and compute histograms and different data statistics.

This thesis is organized as follows. The first two chapters of this work can be seen as a theoretical preamble on Superconducting Transition Edge Sensors and DSP Techniques, the following chapters report the description of the experimental setup and to the characterization, analysis and process of the signal from the TES photo-detector, object of study. In particular, in Chapter 1 a brief introduction to Transition Edge Sensors will be provided including principles of operation, most important figure of merits, and some of the most relevant applications for which are been used nowadays.

Chapter 2 presents a Literature Review on the theory necessary to perform any kind of DSP to the signal object of study: random and stochastic processes theory, signal spectral analysis, and fundamentals on probability and statistical data analysis. In the third Chapter a concise description of the experimental setup used to obtain the experimental data is offered.

Finally, the different parts of the DSP computer-based algorithm are developed and justified by their respective theoretical basis, in the fourth Chapter. Results obtained when processing the signal from the TES photodetector with the corresponding Matlab® routines are presented and discussed along this chapter. To conclude, it is established a comparison between the energy resolution of the TES

detector obtained without any processing of the signal and the one obtained when applying the DSP algorithm.

CHAPTER 1

Transition Edge Sensors

In recent years, superconducting transition-edge sensors (TES) have emerged as outstanding single photons detectors from the near infrared through gamma rays and sensitive detectors of photon fluxes out to millimetre wavelengths. The TESs are thermal sensors that measure an absorbed energy by the increase of resistance of a superconducting film biased within the superconducting to normal transition. This chapter is dedicated to illustrate the basic theory of the superconducting detectors and noise theory and describe practical implementation issues.

1.1 Figure of Merits

When working with single photon detectors it is possible to register single photon absorption events, rather than measuring an optical intensity or power. Photon counting detectors have characteristic properties, also called “figures of merit”, which are simply quantities, used to characterize their performance as single-photon detectors.

The most important figures of merit of all photon detectors are presented next.

- *Sensitivity*. Is defined as the minimal amount of absorbed radiation effectively detected.
- *Dark count rate*. Is the average rate of registered counts without any incident light. This determines the minimum count rate at which the signal is dominantly caused by real photons. The false detection events are mostly of thermal origin and can therefore be strongly suppressed by using a cooled type of detector.
- *Dark Current*. Is the output current which flows when the incident radiation onto the detector is absent or negligible.
- *Time Resolution*. Is a measure of the speed response of the detector. The time resolution is given by the formula: $\tau = 1/2\pi f_c$, where f_c is the 3 dB frequency cut-off¹.
- *Maximum count rate*. Is determined by the speed of the detector or the corresponding electronics. This speed can be limited e.g. by a dead time.
- *Quantum efficiency*. Is the ratio between the number of photons that are actually registered as a valid event and the total number of incidents photons onto the detector. Detection with a small quantum efficiency (i.e., missing out many photons) introduces noise which can be very disturbing.
- *Timing jitter*. As a qualitative term is defined as the uncertainty of the timing of the registered photon events. It is usually quantified with an r.m.s. (root-mean-square) value.
- *Sensitive Band*. Is defined as the wavelength region in which the detector functions properly. It depends on the materials used to create the detector.
- *Energy Resolution*. Is defined as the full width at half maximum of a peak in a spectrum, after subtracting the background under the peak; expressed in units of energy, usually keV; or as a percentage of the energy corresponding to that peak.

¹ Frequency at which the responsivity falls to $\sqrt{1/2} \approx 0.707$ of its maximum value

The energy resolution is a measure of the detector's ability to distinguish closely-spaced lines in the spectrum. Only the detectors with a good intrinsic energy resolution are able to resolve the number of photon of the incident flux. For example, only in superconducting devices, like TES, where the gap energy between the ground state and the exciting state available is in the range of 10-3 eV, can have an intrinsic energy resolution $\Delta E / E \sim 20$.

- *Large-array Adaptability:* In order to have a detector which had also a spatial resolution, it is fundamental that the device can be adapted to use a large number of pixel to form an image.

The figure of merit of different types of photon detectors are reported in Table 1-1

Detector	QE	Time Res.	Sen. Band	Pixels	$\lambda / \Delta\lambda$
Eye[1]	3%	ms - s	400 - 700	10^7	0
Film[1]	4%	s – h	350 - 1100	10^{10}	0
CdS PdS HgCdTg[2]	70%	μ s	250 - 2500		0
MCP-PMT[3]	20-40%	ps	300 - 600	10^7	0
APD[4]	88.2%	Ps	450 - 1100	1	0
CCD[5]	99%	s – min	300 - 1100	10^7	0
STJ	50%	μ s	250 - 2500	10 x 12[6]	~ 22 [7]
TES[8]	80%	μ s	250 - 2500	63	~ 17 [9]

Table 1-1. Comparison of the figures of merit between the different types of detectors in the visible and near-infrared region.

When establishing a comparison between the different detectors and their corresponding figure of merits reported on Table 1-1, it can be understood why the “Cryogenic Innovative Devices” Group of the INRiM has chosen to work on the fabrication and characterization of TESs devices, they have outstanding capabilities as single photon detectors: a high quantum efficiency, a fast speed response (small time resolution), a large sensitivity band and a good intrinsic energy resolution.

1.2 TES Overview

Transition Edge Sensors also called superconducting phase-transition thermometers (SPT), consist of a superconducting film operated in the narrow temperature region between the normal and superconducting state, where the electrical resistance varies between zero and its normal value.

A TES thermometer can be used as a bolometer (to measure power) or as a calorimeter (to measure a pulse of energy).

In 1941, D.H. Andrews applied a current to a fine tantalum wire operating in its superconducting transition region at 3.2K and measured the change in resistance caused by an infrared signal [10]. This was the first demonstration of a TES bolometer. In 1949, the same researcher applied a current to a niobium nitride strip within its superconducting transition at 15K and measured the voltage pulses when it was bombarded by alpha particles [11]. This was the first reported demonstration of a TES calorimeter.

During the first half century after their invention, TES detectors were rarely used in practical applications. One of the principal barriers to their adoption was the difficulty of matching their noise to FET amplifiers (the TES normal resistance is typically a few ohms or less). In recent years, this problem has been largely eliminated by the use of superconducting quantum interference device (SQUID) current amplifiers [12], which are easily impedance-matched to low-resistance TES detectors [13][14]

Another barrier to the practical use of TES detectors has been the difficulty to operate them within the extremely narrow superconducting transition region. When they are current biased, Joule heating of the TES by the current can lead to thermal runaway, and small fluctuations in bath temperature significantly degrade performance. Instead, when the TES is voltage-biased and read out with a current amplifier, the devices can easily be stably biased and they self-regulate in temperature within the transition with much less sensitivity to fluctuations in the bath temperature

[15]. The introduction of voltage-biased operation with SQUID current readout has led to an explosive growth in the development of TES detectors in the past decade.

1.3 Thermal model

TESs are very sensitive thermometers based on the sharp phase transition from the normal to the superconducting state. In Figure 1.1 are shown two typical transition curves, for a Ti and for a TiAu bilayer TES, measured in the Innovative Cryogenic Laboratory. It can be seen from their narrow transition width ($\Delta T_c \sim 2mK$ for Ti TES and $\Delta T_c \sim 1.2mK$ for TiAu TES), that a little variation of temperature corresponds to a great variation of resistance.

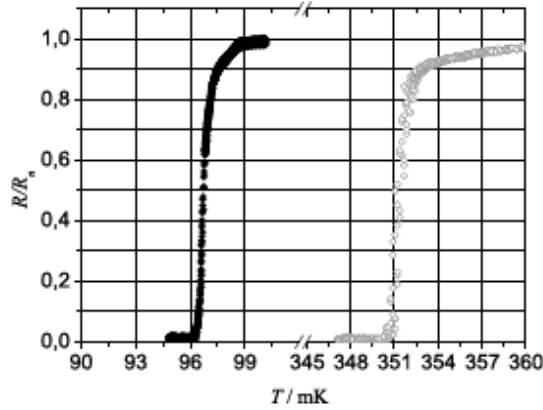


Figure 1.1. The transition curve of a Titanium TES (on the right) and of a TES Ti/Au/Ti multilayer (on the left) normalized to the normal resistance R_n .

In Figure 1.1 is reported the TES thermal model, this model can be referred to also as bilayer TES. At temperatures lower than 1K, from the thermal point of view, the electronic component S_e and the phononic component S_{ph} of a generic superconducting material S with their temperature T_e and T_{ph} , respectively can be distinguish. These two components are thermally connected each other by means of the thermal impedance g_{e-ph} . Another impedance, called g_{S-Sub} connects the phononic component of the superconducting material S_{ph} with the phononic component of the substrate Sub_{ph} . This last one, in its turn, is connected to the heat sink, with a

temperature T_{bath} , by means of the impedance g_{sub} . At the transition temperatures of the devices, $\Delta T_c = 350mK$ for the Ti films and $\Delta T_c = 97mK$ for the TiAu films and $\Delta T_c = 105mK$ for TiPd films, the device is in the electron/phonon decoupling region where the dominating thermal conductance is g_{e-ph} , and all the system inside the dashed block is at the same temperature of the heat sink.

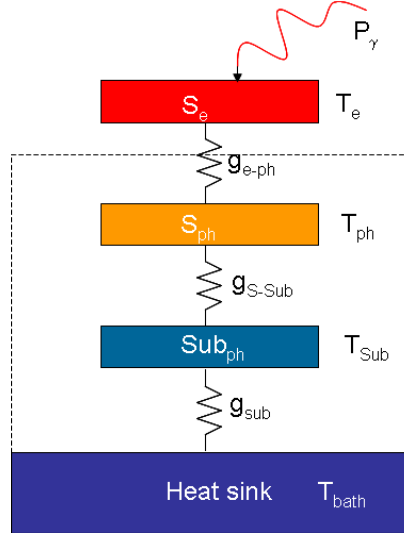


Figure 1.2. The TES thermal model. The electronic component S_e and the phononic component S_{ph} of a generic superconducting material S with their temperature T_e and T_{ph} , respectively can be distinguish. S_e and S_{ph} are thermally connected each other by means of the impedance g_{e-ph} . g_{S-Sub} connects S_{ph} with the phononic component of the substrate Sub_{ph} . This last one, in its turn, is connected to the heat sink, with a temperature T_{bath} , by means of the impedance g_{sub} . Thanks to the electron-phonon decoupling region, all the system inside the dashed block are at the same temperature of the heat sink.

Based on the afore mentioned thermal model, when an incident photon hits the electronic system of the superconducting material, T_e rises before dissipating the heat to the heat sink, through the phonon system S_{ph} . The thermal recovery time, with which the entire system is able to come back to the initial state, is given by the ratio between the heat capacity of the superconducting material and the dominant thermal conductance g_{e-ph} , which will be called G from now on, so we have $\tau_{th} = C_e / G$.

The behaviour of the system can be represented by the power balance equation:

$$C_e \frac{dT(t)}{dt} = P_\gamma - P_{link} \quad (1)$$

where T is the temperature of the TES, P_{link} is the heat loss to the heat sink and P_γ is the power input from the photon. For temperatures approximately equals to T_{bath} , P_{link} is approximated by:

$$P_{link} = G(T(t) - T_{bath}) \quad (2)$$

Substituting P_{link} in Eq. (1) and solving for $T(t)$ produces:

$$T(t) = T_{bath} + \Delta T(t) \quad (3)$$

where,

$$\Delta T(t) = \Delta T_0 e^{-t/\tau_{th}} \quad (4)$$

τ_{th} , as said before, is the time constant governing the exponential decay of the initial temperature rise ΔT_0 , back to T_{bath} . Ideally, the energy is deposited instantaneously leading to an initial temperature rise $\Delta T_0 = E/C$. Consequently, for a known C , the measurement of ΔT gives a direct measurement of the energy deposited in the superconducting material. By measuring the energy of many individual events, it is possible to obtain a spectrum of the source of radiation. Thanks to the biasing method, it is possible to lower this thermal recovery time considering an appropriate electrical circuit with which it is no so important to have a good temperature stability to maintain the device in the middle of its transition curve.

1.4 Electrical model and biasing

In Figure 1.3 there is the electrical circuit used to bias the device and to read out the signal. In the figure, R_{bias} can be distinguish, which is a resistance much lower than

the TES normal state resistance. The input coil L_{in} and the feedback coil L_{fb} , together with the feedback resistor R_{fb} are necessary to use the DC SQUID amplifier in flux-locked loop mode to detect the signal (I_{sensor}).

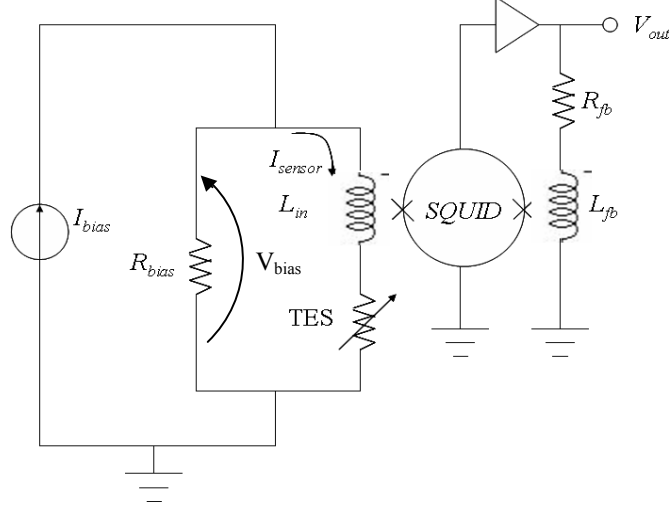


Figure 1.3. TES bias circuit with SQUID readout.

To show how this bias method works, the TES can be consider in its superconducting state. All the bias current I_{bias} passes through the TES ($I_{bias} = I_{sensor}$) until the critical current value is reached. Further increasing of the bias current produces an increase of the TES resistance, in such a way the TES resistance can be controlled by means of the I_{bias} value. The respective dissipated power by the TES is $P_0 = V_{bias}^2 / R_0$, where P_0 and R_0 are the power and the resistance value in the working point, respectively. When an incident photon is absorbed, the TES temperature increases with a consequent increment of the TES resistance, as can be seen in Figure 1.4. This further increase of the TES resistance drive even more I_{bias} throw the R_{bias} , that is ten times lower than the TES normal resistance. In this way, the bias voltage V_{bias} across the parallel between the TES and R_b is fixed until the TES will not return to its superconducting state. Since V_{bias} is as constant voltage and considering the increase of the resistance value, we have a lower value of dissipated power by the TES, and most of the I_{bias} passes through R_{bias} due to the increment of TES resistance. The current I_{sensor} through the device is much lower than before so that TES can come

back in its initial state, ready to detect another event. With this bias model, it is possible to move through the transition of the device without using a temperature control, but simply with an appropriated choice of the bias current I_{bias} (in such a way the temperature of the heat sink $T_{bath} < T_c$ can be maintained without taking too much care of its temperature stability). The bias model explained above is called negative electro-thermal feedback (ETF)[19].

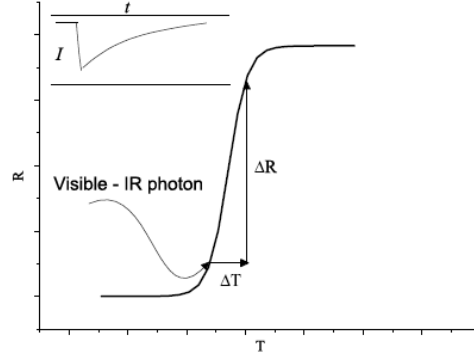


Figure 1.4. Temperature during the transition from the superconducting to the normal region. The inset show a typical current pulse on the output of the detector.

In the inset of the Figure 1.4 it is shown the corresponding negative current pulse measurable using a DC SQUID. In Figure 1.3 we can see the SQUID readout with the input coil in series with the sensor. During the TES transition, the SQUID running in flux-locked mode, gives a voltage feedback signal proportional to the small current variation of I_{sensor} .

The behaviour of the system can be represented by the power balance equation in which the temperature variation caused by the photon incident is considered:

$$C_e \frac{dT(t)}{dt} = \frac{V_{bias}^2}{R(T)} - k(T_e^n - T_{bath}^n) \quad (5)$$

where C_e is the electronic heat capacity of the superconductive material, T_e is the electron temperature of the superconducting material and T_{bath} the heat sink

temperature and k is a constant associated to the material used. Remembering that, for the electron-phonon decoupling, all the elements inside the dashed box of the Figure 1.2 are at the same temperature, consequently T_{bath} is also the temperature of the substrate. Moreover, k is the thermal coupling constant, and n is the thermal coupling power-law exponent in the range of 4-6 depending on the coupling model and on the transition temperature of the superconducting material [20].

If the equilibrium state in which no photon is absorbed is considered the electric power balance the flow of power out of the sample into the bath is given by:

$$\frac{V_{bias}}{R_0} k(T_0^n - T_{bath}^n) \quad (6)$$

where T_0 and R_0 are the steady-state temperature and resistance, respectively.

When an incoming photon with energy E is absorbed, all the energy is very rapidly distributed to the entire electronic system and, as has been already said, a temperature increase of $\Delta T = E/C_e$ is obtained.

When expanding Eq. (5) to the first order in ΔT , we obtain

$$C_e \frac{d\Delta T}{dt} = \frac{V_{bias}^2}{R_0} - k(T_0^n - T_{bath}^n) - \frac{V_{bias}^2}{R_0^2} \frac{dR}{dT} \Delta T - nkT_0^{n-1} \Delta T \quad (7)$$

where n is the thermal coupling power-law exponent with values within the interval $n = [4, 5]$.

From Eq. (6) we can delete the first two terms and setting:

$$G = nkT_0^{n-1} \quad (8)$$

$$\alpha = (T_0 / R_0)(dR / dT) \quad (9)$$

$$P_0 = V_{bias}^2 / R_0 \quad (10)$$

a new differential equation is obtained:

$$\frac{d\Delta T}{dt} = - \left(\frac{P_0}{T_0 C_e} \alpha + \frac{G}{C_e} \right) \Delta T \quad (11)$$

where α gives information on the sharpness of the transition curve of the device and in general it is a positive number $\gg 1$. Usually α is chosen on the 25% of the transition curve [21].

Eq. (11) has a simple exponential solution:

$$\Delta T(t) = \frac{E_\gamma}{C_e} e^{-t/\tau_{eff}} \quad (12)$$

with time constant:

$$\tau_{eff} = C_e \left(\frac{P_0}{T} \alpha + G \right)^{-1} \quad (13)$$

remembering that $\tau_{th} = C_e/G$ is the thermal recovery time without bias power, from Eqs. 6, 8 and 10 it can be obtained:

$$\tau_{eff} = \tau_{th} \left\{ 1 + \frac{\alpha}{n} \left(1 - \frac{T_{bath}^n}{T_0^n} \right)^{-1} \right\} \quad (14)$$

If $T_{bath} \ll T_0$ this corresponds to the strong electrothermal feedback regime and the effective response time τ_{eff} can be approximated in the following way:

$$\tau_{eff} = \frac{n\tau_{th}}{\alpha} \quad (15)$$

It is clear that the effective recovery time τ_{eff} in presence of ETF² bias is much lower than the thermal recovery time τ_{th} with the increasing of α . The bias power, together with the device working point temperature, plays an important role in the energy resolution expression. The intrinsic device resolution is affected by different noise contributions. Between these, for simplicity, only the component relative to the noise intrinsic to the thermal and electrical system will be considered, neglecting noise source arising from the read out system and from the environment. As have been exposed in sec.1.3, the thermal model used is characterized by the separation between the electronic system and the phonon system (and from all the other systems inside

² Electro-Thermal Feedback

the dashed box in Figure 1.2) because of the electron-phonon decoupling region. This separation, although very strong, is not real, because the titanium electron system exhibits temperature fluctuations due to power fluctuations through the thermal conductance to the phonon system. This “phonon noise” can be shown to be the dominant intrinsic noise in a TES with electro-thermal feedback due to the suppression of the Johnson noise [22]. Using the same approximation $\alpha \gg 1$ and $T_c^n \gg T_{bath}^n$ the intrinsic energy resolution is given by:

$$\Delta E_{rms} = \sqrt{\frac{4k_b C_e T_0^2}{\alpha} \sqrt{n/2}} \quad (16)$$

this equation can be expressed using more useful measurable device parameters, so the maximum energy saturation is defined as $E_{sat} = P_0 \tau_{ef}$ which is the maximum energy that the ETF mode can remove during the effective recovery time τ_{ef} .

Remembering Eqs. (15), (6), (8), $\tau_{th} = C_e/G$, and $E_{sat} = C_e T_0 \alpha^{-1}$, Eq. (16) become:

$$\Delta E_{rms} = \sqrt{4k_b T_0 E_{sat} \sqrt{n/2}} \quad (17)$$

From this equation can be understood how the device working point temperature T_0 , plays an important role in the energy resolution determination.

1.5 Applications

The potential of TES detectors is now being realized. TES detectors are being developed for measurements across the electromagnetic spectrum from millimeter [23, 24, 25] through gamma rays [26, 27] as well as with weakly interacting particles [28] and biomolecules [29, 30, 31]. They have contributed to the study of dark matter and supersymmetry [32, 33], the chemical composition of materials [34], and the new field of quantum information [35]. The utility of the single-photon calorimeter have been extended all the way to the near infrared [36], with possible extension to the far infrared; and they are also being used in the first multiplexed submillimeter,

millimeter-wave, and x-ray detectors for spectroscopy and astronomical imaging[36, 37, 24, 25, 38].

CHAPTER 2

Literature Review: DSP Basis

Laplace argued to the effect that given complete knowledge of the physics of an experiment, the outcome must always be predictable. This metaphysical argument must be tempered with several facts. The relevant parameters may not be measurable with sufficient precision due to mechanical or theoretical limits. For this, it is useful to have a theory for the analysis and synthesis of processes that behave in a random or unpredictable manner.

Since the relevant parameters may not be measurable with sufficient precision due to mechanical or theoretical limits, it is useful to have a theory for the analysis and synthesis of processes that behave in a random or unpredictable manner.

The goal of the statistical signal processing theory is to construct mathematical models that lead to reasonably accurate prediction of the long-term average behaviour of random processes, as well as proposing a group of techniques for the manipulation of these signals in order to improved its quality or extract from them only the useful information.

A random or stochastic process is a mathematical model for a phenomenon that evolves in time in an unpredictable manner from the viewpoint of the observer. The

phenomenon may be a sequence of real-valued measurements of voltage or temperature, a binary data stream from a computer, a sequence of images from a cable television, or any of an infinite number of possible sequences, waveforms, or signals of any imaginable type. It may be unpredictable because of effects such as interference or noise in a communication link or storage medium, or it may be an information-bearing signal, deterministic from the viewpoint of an observer at the transmitter but random to an observer at the receiver.

The theory of random processes quantifies the above notions so that it can be possible to construct mathematical models of real phenomena that are both easily manageable and meaningful in the sense of yielding useful predictions of future behaviour. An oversimplified model may provide results and conclusions that do not apply to the real phenomenon being modelled. An overcomplicated one may constrain potential applications, render theory too difficult to be useful, and strain available computational resources.

This section introduces the most important concepts in statistics and probability, with emphasis on how they apply to the acquired signals, along with the basis of some of the most important processing techniques that had been used for the realization of this work.

2.1 Random Variables and Stochastic Processes

Probability Distribution

A probability distribution $p(x)$ may be defined as a non-negative real function of all possible outcomes of some random event. The sum of the probabilities of all possible outcomes is defined as 1, and probabilities can never be negative.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad (18)$$

Random Variable

A random variable x is defined as a real -or complex- valued function of some random event, and is fully characterized by its probability distribution.

Independent Events

Two probabilistic events H_1 and H_2 are said to be independent if the probability of H_1 and H_2 occurring together equals the product of the probabilities of H_1 and H_2 individually, i.e.,

$$p(H_1 H_2) = p(H_1) p(H_2) \quad (19)$$

where $p(H_1 H_2)$ denotes the probability of H_1 and H_2 occurring together.

Stochastic Process

A stochastic process x is defined as a sequence of random variables $x(n)$, $n = \dots, -2, -1, 0, 1, 2, \dots$.

Stationary Stochastic Process

We define a stationary stochastic process $x(n)$, $n = \dots, -2, -1, 0, 1, 2, \dots$, as a stochastic process consisting of identically distributed random variables $x(n)$. In particular, all statistical measures are time-invariant.

Expected Value

The expected value of a continuous random variable $\nu \in (-\infty, \infty)$ is denoted $E\{\nu\}$ and is defined by

$$E\{\nu\} = \int_{-\infty}^{\infty} x f_{\nu}(x) dx \quad (20)$$

where $p_{\nu}(x)$ denotes the probability density function (PDF) for the random variable ν .

Mean

The mean of a stochastic process $\nu(n)$ at time n is defined as its expected value:

$$\mu_{\nu(n)} = E\{\nu(n)\} = \int_{-\infty}^{\infty} x p_{\nu(n)}(x) dx \quad (21)$$

where $p_{\nu(n)}(x)$ is the probability density function for the random variable $\nu(n)$.

For a stationary stochastic process ν , the mean is given by the expected value of $\nu(n)$ for any n. I.e., $\mu_{\nu} = E\{\nu(n)\}$ for all n.

Sample Mean.

The sample mean of a set of N samples from a particular realization of a stationary stochastic process ν is defined as the average of those samples:

$$\mu_{\nu} = \varepsilon_N \{\nu(0 : N-1)\} = \frac{1}{N} \sum_{n=0}^{N-1} \nu(n) \quad (22)$$

For a stationary stochastic process ν , the sample mean is an unbiased estimator of the mean, i.e., $E\{\hat{\mu}_\nu\} = \mu_\nu$

Variance

The variance or second central moment of a stochastic process $\nu(n)$ at time n is defined as the expected value of $|\nu(n) - \mu_{\nu(n)}|^2$:

$$\sigma_{\nu(n)}^2 = E\left\{|\nu(n) - \mu_{\nu(n)}|^2\right\} = \int_{-\infty}^{\infty} |\nu(n) - \mu_{\nu(n)}|^2 p_{\nu(n)}(x) dx \quad (23)$$

where $p_{\nu(n)}(x)$ is the probability density function for the random variable $\nu(n)$.

For a stationary stochastic process ν , the variance is given by the expected value of $|\nu(n) - \mu_\nu|^2$ for any n .

Sample Variance

The sample variance of a set of N samples from a particular realization of a stationary stochastic process ν is defined as average squared magnitude after removing the known mean:

$$\hat{\sigma}_\nu^2 = \varepsilon_N \left\{|\nu(n) - \mu_\nu|^2\right\} = \frac{1}{N} \sum_{n=0}^{N-1} |\nu(n) - \mu_\nu|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |\nu(n)|^2 - \mu_\nu^2 \quad (24)$$

The sample variance is an unbiased estimator of the true variance when the mean is known, i.e., $E\{\hat{\sigma}_\nu^2\} = \sigma_\nu^2$

This is easy to show by taking the expected value:

$$\begin{aligned}
E\{\hat{\sigma}_v^2\} &= E\left[\varepsilon_N\left\{|\nu(n) - \mu_v|^2\right\}\right] = \varepsilon_N\left\{E|\nu(n) - \mu_v|^2\right\} \\
&= \varepsilon_N\left\{E|\nu(n)|^2 - E(\bar{\nu}(n)\mu_v) - E(\nu(n)\bar{\mu}_v) - |\mu_v|^2\right\} \\
&= \varepsilon_N\left\{\hat{\sigma}_v^2 + |\mu_v|^2 - \bar{\mu}_v\mu_v - \mu_v\bar{\mu}_v - |\mu_v|^2\right\} \\
&= \varepsilon_N\{\sigma_v^2\} = \sigma_v^2
\end{aligned} \tag{25}$$

When the mean is unknown, the sample mean is used in its place:

$$\hat{\sigma}_v^2 = \frac{1}{N-1} \sum_{n=0}^{N-1} |\nu(n) - \hat{\mu}_v|^2 \tag{26}$$

The normalization by $N-1$ instead of N is necessary to make the sample variance be an unbiased estimator of the true variance. This adjustment is necessary because the sample mean is correlated with the term $\nu(n)$ in the sample variance expression.

2.2 Spectral Analysis

Signal processing involves techniques that improve the understanding of information contained in experimental data. Normally, when a signal is measured it is studied in the time domain. For many signals, this is the most logical and intuitive way to view them.

When the frequency content of the signal is of interest, it makes sense to study it in the frequency domain. In the frequency domain it can be seen how much of the signal's energy is present as a function of frequency.

One of the important tools in signal processing is spectral analysis: examining the frequency content in a signal. Generally, this is done by using the discrete Fourier transform.

When referring to a real signal, spectral estimation is used to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data.

The discrete Fourier transform (DFT) maps a finite number of discrete time-domain samples to the same number of discrete Fourier-domain samples. Being practical to compute, it is the primary transform applied to real-world sampled data in digital signal processing.

The DFT has special relationships with the discrete-time Fourier transform and the continuous-time Fourier transform that let it be used as a practical approximation of them through truncation and windowing of an infinite-length signal. Different window functions make various tradeoffs in the spectral distortions and artifacts introduced by DFT-based spectrum analysis. The next section is dedicated to the discussion of the most important concepts on the Fourier Theory.

2.2.1 Fourier Transform

The behaviour of a system can be mathematically modelled and represented both in the time domain and in the frequency domain. Fourier Transform permits mapping time domain functions into frequency domain representations.

The importance of Fourier transform lies on the fact that it allows studying the frequency content of signals, their spectral properties and the relationship existing between their spectra and time domain characteristics. In order to be able to perform the spectral analysis of a signal it is necessary to understand some of the basic concepts of the Fourier Theory.

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega)$$

The Fourier Transform is defined as:

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \quad (27)$$

where $x(t)$ is the time domain signal, and $X(\omega)$ is its Fourier Transform. With the inverse Fourier transform, the original signal is given by:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega \quad (28)$$

Existence of the Fourier Transform

Conditions for the existence of the Fourier transform are complicated to state in general, but it is sufficient for $x(t)$ to be absolutely integrable, i.e.,

$$\|x\|_1 = \int_{-\infty}^{\infty} |x(t)| dt < \infty \quad (29)$$

This requirement can be stated as $x \in L_1$, meaning that x belongs to the set of all signals having a finite L_1 norm ($\|x\|_1 < \infty$). It is similarly sufficient for $x(t)$ to be square integrable, i.e.,

$$\|x\|_2^2 = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty \quad (30)$$

or, $x \in L_2$. More generally, it suffices to show $x \in L_p$ for $1 \leq p \leq 2$.

There is never a question of existence, of course, for Fourier transforms of real-world signals encountered in practice. However, idealized signals, such as sinusoids that go on forever in time, do pose normalization difficulties. In practical engineering analysis, these difficulties are resolved using Dirac's “generalized functions” such as the impulse (also called the delta function)

2.2.2 Discrete Fourier Transform

Similarly, the discrete Fourier transform (DFT) maps discrete time sequences into discrete frequency representations. The sequence of N complex numbers $x_0, \dots, x_{N-1}$ is transformed into the sequence of N complex numbers $X_0, \dots, X_{N-1}$ by the DFT according to the formula:

$$x_k = \sum_{i=0}^{n-1} x_i e^{-j2\pi i k / n} \quad \text{for } k = 0, 1, 2, \dots, n-1 \quad (31)$$

where x is the input sequence, X is its DFT, and n is the number of samples in both the discrete time and the discrete frequency domains.

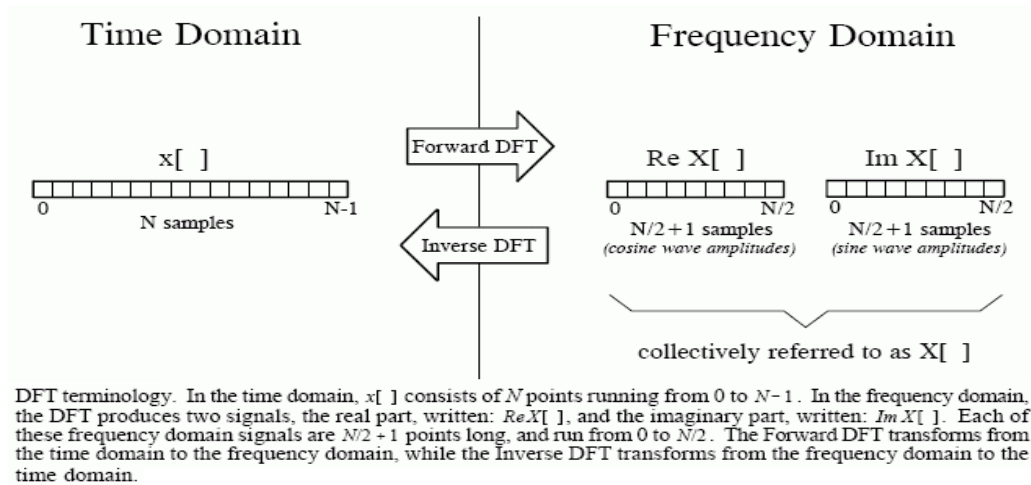


Figure 2.1. Discrete Fourier Transform

2.2.3 Applications of the DFT

The Discrete Fourier Transform (DFT) is one of the most important tools in Signal Processing. The DFT can calculate the frequency spectrum of a real signal that has been sampled. This is a direct examination of information encoded in the frequency,

phase, and amplitude of the component sinusoids. This allows systems to be analyzed in the frequency domain.

The DFT can be used as an intermediate step in more elaborate signal processing techniques. The classic example of this is FFT (fast Fourier transform) convolution, an algorithm for convolving signals that is hundreds of times faster than conventional methods.

In the digital signal processing area, the DFT has three main applications:

1. Fast calculation of the convolution and the correlation functions
2. Spectral analysis of digital signals, with use in the synthesis of digital filters
3. Data compression for its memorization and efficient transmission

Convolution

A very important concept in Fourier theory is that of a convolution. Because of a mathematical property of the Fourier transform, referred to as the convolution theorem, it is convenient to carry out calculations involving convolutions.

The convolution of two complex signals, f and g is written $f * g$. It is defined as the integral over time of the product of the two functions after one is reversed and shifted.

$$\begin{aligned} f(t) * g(t) &= \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau && \text{(Commutativity)} \\ &= \int_{-\infty}^{\infty} f(t - \tau) \cdot g(\tau) d\tau && (32) \end{aligned}$$

In the same way, for complex-valued functions f, g , the *discrete convolution* of f and g is given by:

$$\begin{aligned}
f[n] * g[n] &= \sum_{m=-\infty}^{\infty} f[m] \cdot g[m-n] && \text{(Commutativity)} \\
&= \sum_{m=-\infty}^{\infty} f[n-m] \cdot g[m] && (33)
\end{aligned}$$

Convolution Theorem

The convolution theorem states that under suitable conditions the Fourier transform of a convolution is the pointwise product of Fourier transforms. In other words, convolution in one domain (e.g., time domain) equals point-wise multiplication in the other domain (e.g., frequency domain). Let f and g be two functions with convolution $f * g$. Let F denote the Fourier transform operator, so $F\{f\}$ and $F\{g\}$ are the Fourier transforms of f and g , respectively. Then:

$$F\{f * g\} = F\{f\} \cdot F\{g\} \quad (34)$$

where $\cdot$ denotes point-wise multiplication. It also works the other way around:

$$F\{f \cdot g\} = F\{f\} * F\{g\} \quad (35)$$

By applying the inverse Fourier transform F^{-1} , can be written:

$$f * g = F^{-1}\{F\{f\} \cdot F\{g\}\} \quad (36)$$

This formulation is especially useful for implementing a numerical convolution on a computer: The standard convolution algorithm has quadratic computational complexity. With the help of the convolution theorem and a fast Fourier transform -which is an efficient algorithm to compute the discrete Fourier transform (DFT) and its inverse-, the complexity of the convolution can be reduced to

$O(n \log n)$ instead of $O(N^2)$. This has been exploited to construct fast multiplication algorithms.

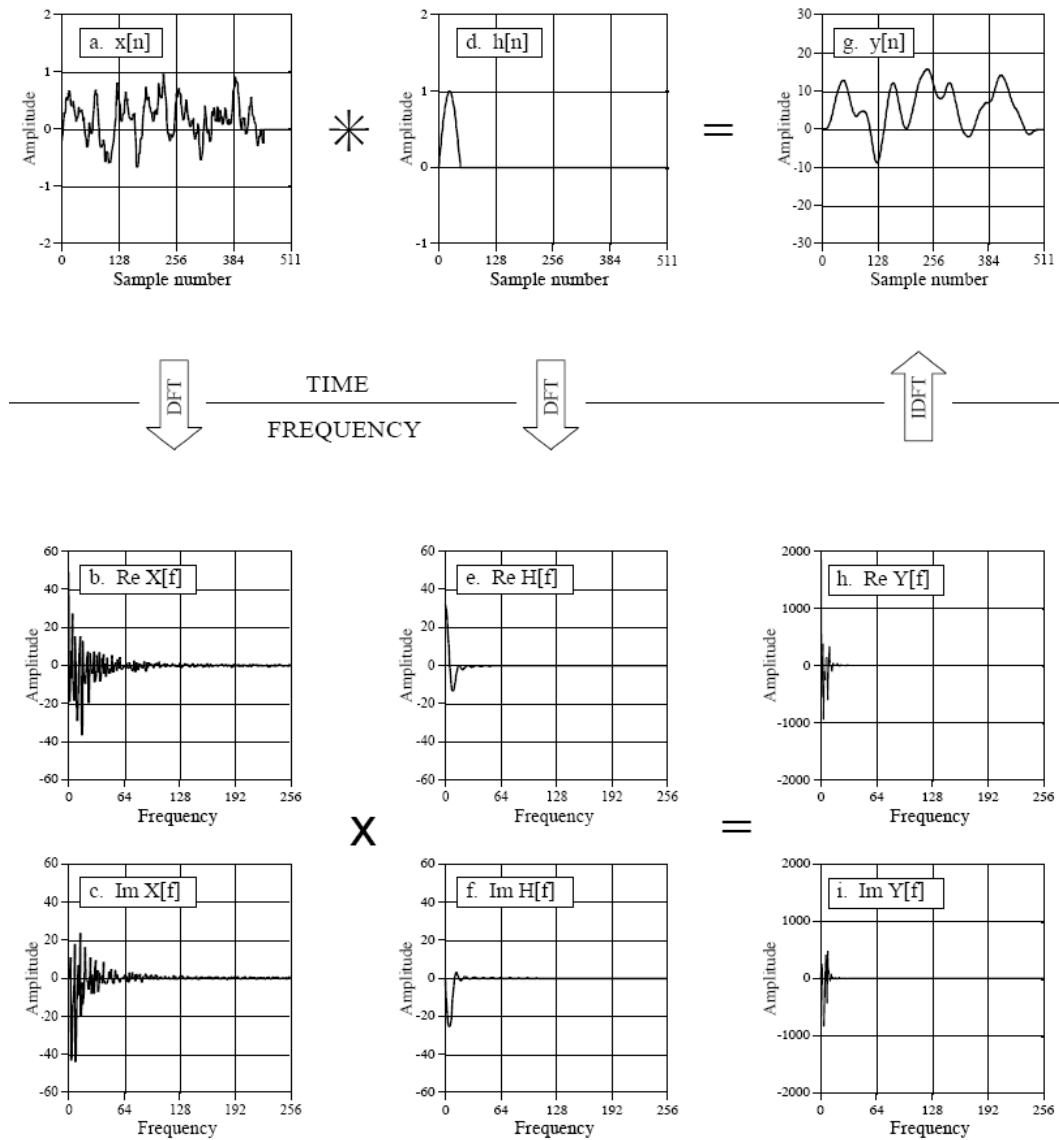


Figure 2.2. Frequency domain convolution. In the *time domain*, $x[n]$ is convolved with $h[n]$ resulting in $y[n]$, as is shown in Figs. (a), (d), and (g). This same procedure to be accomplished in the *frequency domain*. The DFT is used to find the frequency spectrum of the input signal, (b) & (c), and the system's frequency response, (e) & (f). Multiplying these two frequency domain signals results in the frequency spectrum of the output signal, (h) & (i). The Inverse DFT is then used to find the output signal, (g).

Correlation Analysis

Correlation is another important concept of the Fourier analysis. It gives an idea of the degree of linear association between two statistical variables. Worth mention that correlation analysis applies only to stationary stochastic processes. Next, the most important concepts on correlation analysis will be explained.

Cross-Correlation

The cross-correlation of two signals x and y may be defined by:

$$r_{xy}(l) = E \{ \bar{x}(n)y(n+l) \} \quad (37)$$

I.e., it is the expected value of the lagged products in random signals x and y . In principle, the expected value must be computed by averaging $x(n)y(n+l)$ over many realizations of the stochastic process x and y .

If the signals are stationary (which means their statistics are time-invariant), then we may replace ensemble averaging by averaging across time. In other words, as has been said before, for stationary stochastic processes, time averages equal ensemble averages.

Cross-Power Spectral Density

The Discrete Time Fourier Transform (DTFT) of the cross-correlation is called the cross-power spectral density, or “cross-spectral density”, “cross-power spectrum”, or even simply “cross-spectrum”.

Autocorrelation

The cross-correlation of a signal with itself gives the autocorrelation function of that signal:

$$r_x(l) = r_{xx}(l) = E\{\bar{x}(n)x(n+l)\} \quad (38)$$

Note that the autocorrelation function is Hermitian³:

$$r_x(-l) = \bar{r}_x(l) \quad (39)$$

When x is real, its autocorrelation is symmetric. More specifically, it is real and even.

Sample Autocorrelation.

The sample autocorrelation of a sequence $v(n)$, $n = 0, 1, 2, \dots, N-1$ may be defined by

$$\hat{r}_{v,N}(l) = \frac{1}{N-|l|} \sum_{n=0}^{N-1} \bar{v}(n)v(n+l) \quad l = 0, \pm 1, \pm 2, \dots, \pm(N-1) \quad (40)$$

where $v(n)$ is defined as zero outside of the range $n \in [0, N-1]$.

Power Spectral Density

The goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise.

³ A Hermitian function is a complex function with the property that its complex conjugate is equal to the original function with the variable changed in sign.

In general, the Fourier transform of the autocorrelation function $r_x(l)$ is called the power spectral density (PSD), or power spectrum, and may be denoted as:

$$S_{xx}(\omega) = DTFT_{\omega}(r_{xx}) \quad (41)$$

In terms of normalized frequency, the power spectrum of a stationary random process x_n is mathematically related to the correlation sequence, by the discrete-time Fourier transform, through the following expression:

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} r_{xx}(m) e^{-j\omega m} \quad (42)$$

This can be written as a function of physical frequency f (e.g., in hertz) by using the relation $\omega = 2\pi f / f_s$, where f_s is the sampling frequency.

$$S_{xx}(f) = \sum_{m=-\infty}^{\infty} r_{xx}(m) e^{-2\pi jmf / f_s} \quad (43)$$

The correlation sequence can be derived from the power spectrum by use of the inverse discrete-time Fourier transform:

$$r_{xx}(m) = \int_{-\pi}^{\pi} \frac{S_{xx}(\omega) e^{j\omega m}}{2\pi} d\omega = \int_{-f_s/2}^{f_s/2} \frac{S_{xx}(f) e^{2\pi jmf / f_s}}{f_s} df \quad (44)$$

The average power of the sequence x_n over the entire Nyquist interval is represented by:

$$r_{xx}(0) = \int_{-\pi}^{\pi} \frac{S_{xx}(\omega)}{2\pi} d\omega = \int_{-f_s/2}^{f_s/2} \frac{S_{xx}(f)}{f_s} df \quad (45)$$

The quantities

$$P_{xx}(\omega) = \frac{S_{xx}(\omega)}{2\pi} \quad \text{and} \quad P_{xx}(f) = \frac{S_{xx}(f)}{f_s} \quad (46)$$

from the above expression are defined as the power spectral density (PSD) of the stationary random signal x_n .

When the signal x is real, its PSD is real and even, like its autocorrelation function.

Sample Power Spectral Density

The Fourier transform of the sample autocorrelation function $\hat{r}_{v,N}$ (see equation (22)) is defined as the sample power spectral density (PSD):

$$\hat{S}_{v,N}(\omega) = DTFT_{\omega} \{ \hat{r}_{v,N} \} = \sum_{n=-\infty}^{\infty} \hat{r}_{v,N}(n) e^{-j\omega n} \quad (47)$$

As mentioned before, for real signals, the autocorrelation function is always real and even, and therefore the power spectral density is real and even for all real signals. The PSD $S_x(\omega)$ can be interpreted as a measure of the relative probability that the signal contains energy at frequency ω in a window centered at a given arbitrary time. An area under the PSD, $S_x(\omega) \cdot \Delta\omega$, comprises the contribution to the variance of $x(n)$ from the frequency interval $[\omega, \omega + \Delta\omega]$. The total integral of the PSD gives the total variance:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} S_v(\omega) d\omega = r_v(0) = \sigma_v^2 \quad (48)$$

again assuming $v(n)$ is zero mean.

Correlation Theorem

The correlation theorem is closely related to the convolution theorem, and it is also very useful in many computations. The correlation theorem is a result that applies to the correlation function, which is an integral that has a definition reminiscent of the convolution integral.

$$F\{r_{xy}(t)\} = \bar{X}(\omega)Y(\omega) \quad (49)$$

Frequency Response of Systems

Systems are analyzed in the *time domain* by using convolution. A similar analysis can be done in the *frequency domain*. Using the Fourier transform, every input signal can be represented as a group of cosine waves, each with a specified amplitude and phase shift. Likewise, the DFT can be used to represent every output signal in a similar form. This means that any linear system can be *completely* described by how it changes the amplitude and phase of cosine waves passing through it. This information is called the system's **frequency response**. Since both the impulse response and the frequency response contain complete information about the system, there must be a one-to-one correspondence between the two of them. Given one, you can calculate the other. The relationship between the impulse response and the frequency response is one of the foundations of signal processing: *A system's frequency response is the Fourier Transform of its impulse response*⁴. Figure 2.3 illustrates these relationships.

Standard notation, uses lower case variables for impulse responses, while the corresponding frequency responses are upper case. Since $h[]$ is the common symbol for the impulse response, $H[]$ is used for the frequency response.

⁴ The impulse response of a digital filter is the output arising from the unit impulse input sequence defined as $x(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$.

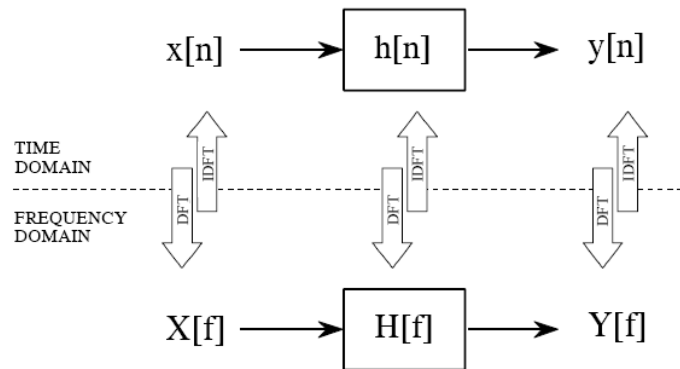


Figure 2.3. Comparing system operation in the time and frequency domains. In the time domain, an input signal is *convolved* with an impulse response, resulting in the output signal, that is, $x[n] * h[n] = y[n]$. In the frequency domain, an input spectrum is *multiplied* by a frequency response, resulting in the output spectrum, that is, $X[f] \times H[f] = Y[f]$. The DFT and the Inverse DFT relate the signals in the two domains.

Systems are described in the time domain by $x[n] * h[n] = y[n]$. Where $x[n]$ represents the input signal and $y[n]$ represents the output of the system. Applying the convolution theorem, in the frequency domain, the system is represented as $X[f] \cdot H[f] = Y[f]$.

Figure 2.4 shows an example of using the DFT to convert a system's impulse response into its frequency response. Figure (a) is the impulse response of the system. Taking a 64 point DFT of this impulse response produces the frequency response of the system, shown in (b).

Figure (b) is very jagged due to the low number of samples defining the curve. This situation can be improved by padding the impulse response with zeros before taking the DFT. For example, adding zeros to make the impulse response 512 samples long, as shown in (c), results in the higher resolution frequency response shown in (d).

Even though the impulse response is a *discrete* signal, the corresponding frequency response is *continuous*. An N point DFT of the impulse response provides $N/2 + 1$ samples of this continuous curve. Making the DFT longer, improves the

resolution of the frequency responses, which are the amplitude and phase changes experienced by cosine waves as they pass through the system.

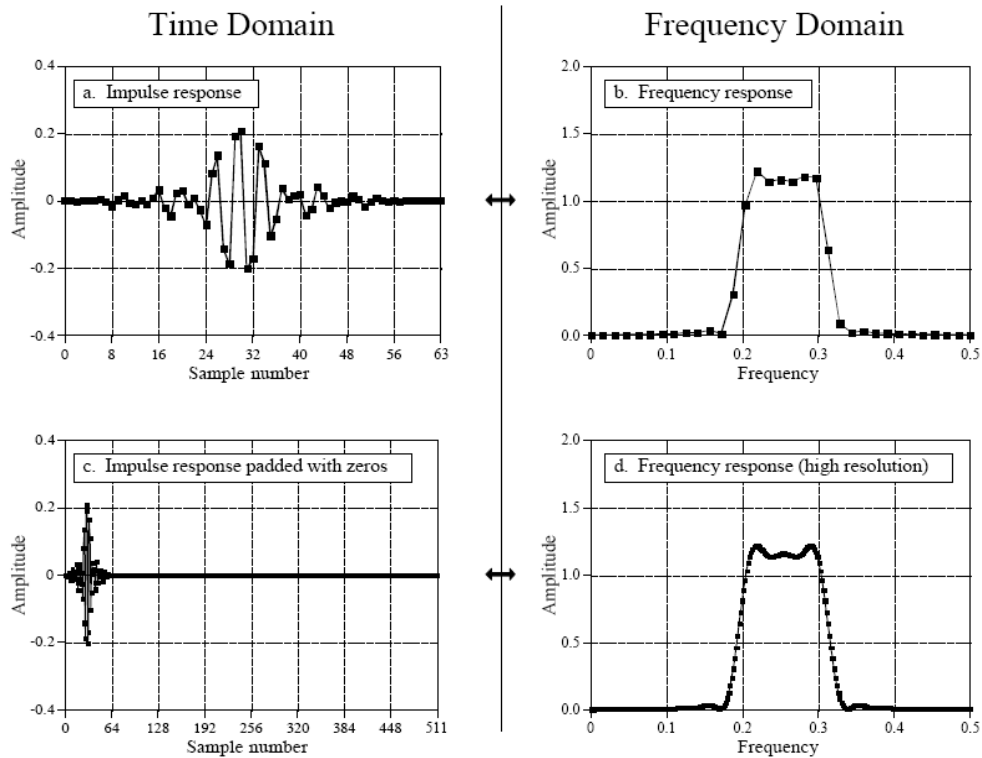


Figure 2.4. Finding the frequency response from the impulse response. By using the DFT, a system's impulse response, (a), can be transformed into the system's frequency response, (b). By padding the impulse response with zeros (c), higher resolution can be obtained in the frequency response, (d). Only the magnitude of the frequency response is shown in this example

Digital Filters

By utilizing DSP techniques capable of sequencing and reproducing hundreds to thousands of discrete elements, large hardware structures can be simulated at relatively low cost. DSP techniques can perform functions such as Fast-Fourier Transforms (FFT), delay equalization, programmable gain, modulation, encoding/decoding, and filtering. Programs can be written where filter weighting

functions (coefficients) can be calculated on the fly, reducing memory requirements or algorithms can be dynamically modified as a function of signal input.

A digital filter computes a quantized time-domain representation of the convolution of the sampled input time function and a representation of the weighting function of the filter. They are realized by an extended sequence of multiplications and additions carried out at a uniformly spaced sample interval. Simply said, the digitized input signal is mathematically influenced by the DSP program. These signals are passed through structures that shift the clocked data into summers (adders), delay blocks and multipliers. These structures change the mathematical values in a predetermined way; the resulting data represents the filtered or transformed signal.

It is important to note that distortion and noise can be introduced into digital filters simply by the conversion of analogue signals into digital data and by the digital filtering process itself.

Digital filters are used for two general purposes: separation of signals that have been combined, and restoration of signals that have been distorted in some way. Analogue (electronic) filters can be used for these same tasks; however, digital filters can achieve far superior results.

Signal separation is needed when a signal has been contaminated with interference, noise, or other signals. Signal restoration is used when a signal has been distorted in some way. Both analogue and digital filters can be used. While analogue filters are cheaper, faster, and have a larger dynamic range in both amplitude and frequency, digital filters, are vastly superior in the level of performance that can be achieved.

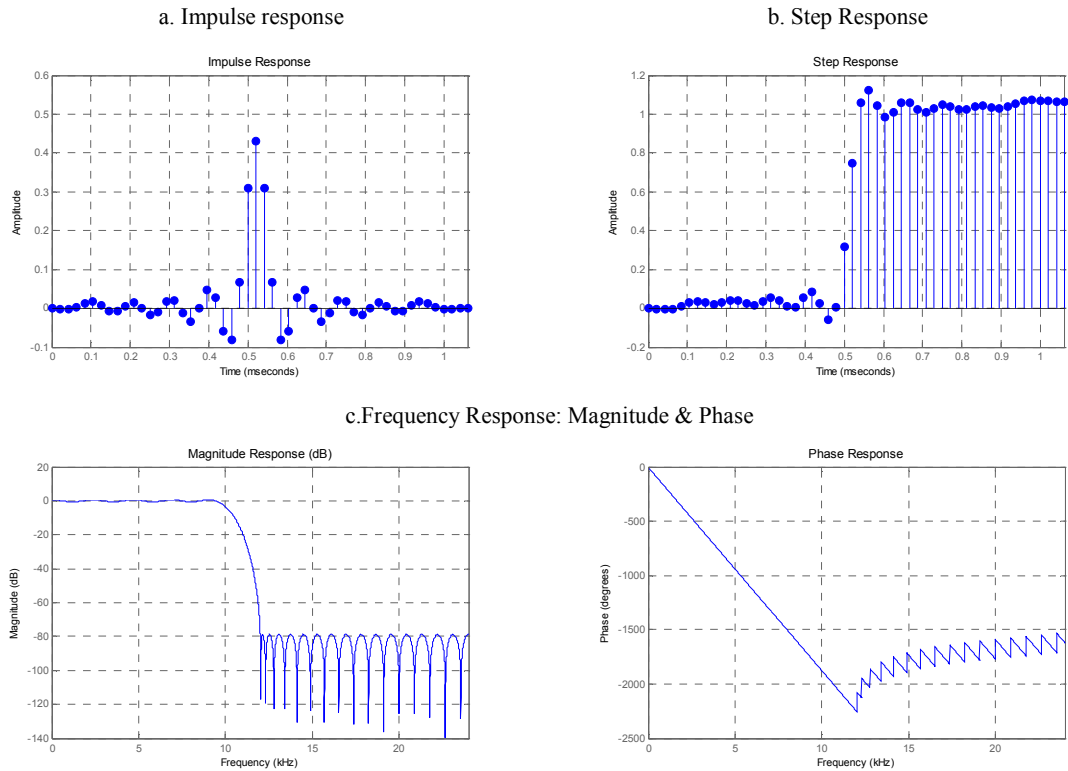


Figure 2.5: Filter responses. Every linear filter has an impulse response, a step response, and a frequency response. The step response, (b), can be found by discrete integration of the impulse response, (a). The frequency response can be found from the impulse response by using the Fast Fourier Transform (FFT), and can be displayed either on a linear scale, or in decibels (c).

As mention before, every linear filter can be defined by his impulse response, step response or frequency response. Each of these responses contains complete information about the filter. If one of the three is specified, the other two are fixed and can be directly calculated. All three of these representations are important, because they describe how the filter will react under different circumstances.

The most straightforward way to implement a digital filter is by *convolving* the input signal with the digital filter's *impulse response*, applying the convolution theorem. All possible linear filters -and systems- can be made in this manner.

There are two types of filters in the digital realm, according to the technique employed to design them. These are: Infinite Impulse Response (IIR) filters, and Finite Impulse Response (FIR) filters.

FIR “Finite Impulse Response” Implementations

FIR (Finite Impulse Response) filters are implemented using a finite number " n " of delay blocks on a delay line and " n " computation coefficients to compute the algorithm function. The bellow structure is a non-recursive repetitive delay-and-add format, and is most often used to produce FIR filters. This structure depends upon each sample of new and present value data.

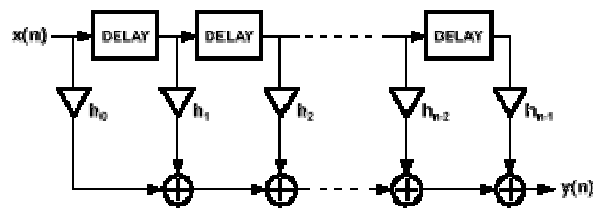


Figure 2.6. Direct Form-I Structure of an FIR Digital Filter

The majority of the DSP applications employ Fixed-Point⁵ DSP processors because of their smaller size and lower cost. The Fixed-Point math requires programmers to pay significant attention to the number of coefficients utilized in each algorithm when multiplying and accumulating digital data to prevent distortion caused by register overflow and a decrease of the signal-to-noise ratio caused by truncation noise. The structure of these algorithms uses a repetitive delay-and-add format that can be represented as "Direct Form-I Structure" like the one shown in Figure 2.6

FIR filters can create transfer functions that have no equivalent in linear circuit technology. They can offer shape factor accuracy and stability equivalent to very

⁵ A fixed-point number representation is a real data type for a number that has a fixed number of digits after (and sometimes also before) the radix point.

high-order linear active filters that cannot be achieved in the analogue domain. Unlike IIR (Infinite Impulse Response) filters (See Section 0 below), FIR filters are formed with only the equivalent of zeros in the linear domain. This means that the delay blocks lower the amplitude of the transfer function. The attenuation of the amplitude transfer function depends upon the value of the multiplier coefficient. Hence, the total number of delays determines the sharpness of the filter roll-off. This can be inferred from the structure shown in Figure 2.6 above.

The number of delays and the values of the computation coefficients ($h_0, h_1, \dots, h_n$) are selected to "weight" the data being shifted down the delay line to create the desired amplitude response of the filter.

In this configuration there are no feedback paths to cause instability. The calculation coefficients are not constrained to particular values and can be used to implement filter functions that do not have a linear system equivalent. Note that more delays increase the steepness of the filter roll-off while increasing calculation time, and for high order filters, while limiting bandwidth.

When designing a FIR filter attention must be paid to the tradeoffs between phase delay and filter precision. The disadvantage is that high order FIR filters have longer delay; while the advantage is that the phase response remains linear as a function of frequency. In applications where linear phase is critical and long phase delay cannot be tolerated, a linear active Bessel or a constant delay filter may be a better selection.

Two design techniques are commonly used to develop digital FIR filters: the Window Technique and the Equiripple Technique. In this work, only the window technique will be used.

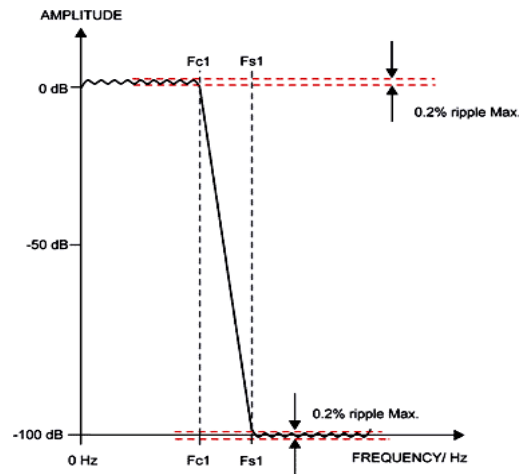


Figure 2.7. Low-pass FIR Filter Template

IIR “Infinite Impulse Response” Implementations

The equivalent of classical linear-system transfer functions can be implemented by using IIR implementation techniques. A common procedure is to start with the classic analogue filter transfer function, such as a Butterworth, and apply the required transform to convert the filter equations from the complex S-domain to the complex Z-domain. The resulting coefficients yield a Z-domain transfer function in a feedback configuration with a number " n " of delay nodes that is equal to the order of the S-domain transfer function. These implementations are referred to as IIR filters because when a short impulse is put through the filter, the output value does not converge quickly to zero, but theoretically continues decreasing over an infinite number of samples.

Floating Point DSP's can produce near equivalent analogue filter transforms such as Butterworth, Chebyshev and elliptic because they use essentially the same mathematical structure as their analogue counterparts. For the same reason, they exhibit the same or worse non-linear phase characteristics, since the equivalent of poles and zeros in linear systems are reproduced with an IIR, digital filter.

DSP's can perform floating-point math, which greatly decreases truncation noise problems and allows more complicated filter structures such as the inclusion of both poles and zeros. This permits the approximation of many waveforms or transfer

functions that can be expressed as an infinite recursive series, such as Infinite Impulse Response (IIR) filters. The functions are infinite recursive because they use previously calculated values in future calculations analogous to feedback in hardware systems.

Figure 2.8 illustrates a bi-quad digital filter structure that computes the response of a second order IIR transfer function. It has two delay nodes and the computation coefficients are A_{1k} , A_{2k} , B_{1k} and B_{2k} .

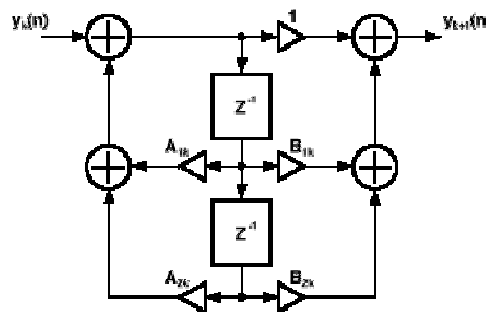


Figure 2.8. Bi-quad Digital Filter that Computes Second Order IIR Transfer Function

Comparison between FIR and IIR Filters

Compared to IIR filters, FIR filters offer the following advantages:

- They can easily be designed to be "linear phase" (and usually are). Put simply, linear-phase filters delay the input signal, but don't distort its phase.
- They are simple to implement. On most DSP microprocessors, the FIR calculation can be done by looping a single instruction.
- They are suited to multi-rate applications. By multi-rate, we mean either "decimation" (reducing the sampling rate), "interpolation" (increasing the sampling rate), or both. Whether decimating or interpolating, the use of FIR filters allows some of the calculations to be omitted, thus providing an important computational efficiency. In contrast, if IIR filters are used, each output must be individually calculated, even if it that output will be discarded (so the feedback will be incorporated into the filter).

- They have desirable numeric properties. In practice, all DSP filters must be implemented using "finite-precision" arithmetic, that is, a limited number of bits. The use of finite-precision arithmetic in IIR filters can cause significant problems due to the use of feedback, but FIR filters have no feedback, so they can usually be implemented using fewer bits, and the designer has fewer practical problems to solve related to non-ideal arithmetic.
- They can be implemented using fractional arithmetic. Unlike IIR filters, it is always possible to implement a FIR filter using coefficients with magnitude of less than 1.0. (The overall gain of the FIR filter can be adjusted at its output, if desired.) This is an important consideration when using fixed-point DSP's, because it makes the implementation much simpler.

On the other hand, FIR filters sometimes have the disadvantage that they require more memory and/or calculation to achieve a given filter response characteristic; and naturally certain responses are not practical to implement with FIR filters.

As said before, in this thesis no circuitry was constructed, however the DSP routine that is proposed in this work had been developed with the idea of a possible future implementation. For this reason, and for all the mentioned before in reference to the advantages of the FIR filters over the IIR filters, the digital filters that will be presented on this section are of the FIR type constructed with a window technique.

Matched Filter

A *matched filter* is obtained by correlating a known signal, or template, with an unknown signal to detect the presence of the template in the unknown signal. This is equivalent to convolving the unknown signal with a time-reversed version of the template (cross-correlation). The matched filter is the optimal linear filter for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise.

The matched filter is the linear filter, h , that maximizes the output signal-to-noise ratio.

The observed signal consists of the desirable signal s and additive noise v :

$$x = s + v \quad (50)$$

The covariance matrix of the noise is defined as:

$$R_v = E\{vv^H\} \quad (51)$$

where $(\cdot)^H$ denotes Hermitian (conjugate) transpose, and E denotes expectation.

The inner product of the filter and the observed signal will be defined as y , such that

$$y = \sum_{k=-\infty}^{\infty} h^*[k]x[k] = h^H x = h^H s + h^H v = y_s + y_v \quad (52)$$

The signal-to-noise ratio is defined as:

$$SNR = \frac{|y_s|^2}{E\{|y_v|^2\}} \quad (53)$$

Rewriting the expression above:

$$SNR = \frac{|h^H s|^2}{E\{|h^H v|^2\}} \quad (54)$$

The objective is to maximize this quantity by choosing h , expanding the denominator of the SNR ratio the following expression is obtained:

$$E\{|h^H v|^2\} = E\{(h^H v)(h^H v)^H\} = h^H E\{vv^H\} h = h^H R_v h \quad (55)$$

Now, the SNR becomes

$$SNR = \frac{|h^H s|^2}{h^H R_v h} \quad (56)$$

This expression will be rewrite with some matrix manipulation. The reason for this seemingly counterproductive measure will become evident shortly.

Exploiting the Hermitian symmetry of the covariance matrix R_v , it can be wrote:

$$SNR = \frac{|(R_v^{1/2}h)^H (R_v^{-1/2}s)|^2}{(R_v^{1/2}h)^H (R_v^{1/2}h)} \quad (57)$$

To define an upper bound to this expression, first a form of the Cauchy-Schwarz inequality will be recalled:

$$|a^H b|^2 \leq (a^H a)(b^H b) \quad (58)$$

The square of the inner product of two vectors can only be as large as the product of the individual inner products of the vectors. This concept returns to the intuition behind the matched filter: this upper bound is achieved when the two vectors a and b are parallel.

The derivation of the matched filter is resumed by expressing the upper bound on our SNR in light of the geometric inequality above:

$$SNR = \frac{|(R_v^{1/2}h)^H (R_v^{-1/2}s)|^2}{(R_v^{1/2}h)^H (R_v^{1/2}h)} \leq \frac{[(R_v^{1/2}h)^H (R_v^{1/2}h)][(R_v^{-1/2}s)^H (R_v^{-1/2}s)]}{(R_v^{1/2}h)^H (R_v^{1/2}h)} \quad (59)$$

It can be seen that the expression for the upper bound can be greatly simplified:

$$SNR = \frac{|(R_v^{1/2}h)^H (R_v^{-1/2}s)|^2}{(R_v^{1/2}h)^H (R_v^{1/2}h)} \leq s^H R_v^{-1} s \quad (60)$$

The upper bound can be achieved if choosing:

$$R_v^{1/2}h = \alpha R_v^{1/2}s \quad (61)$$

where α is an arbitrary real number.

To verify this, we replace this into the expression of the SNR :

$$SNR = \frac{|(R_v^{-1/2}h)^H (R_v^{-1/2}s)|^2}{(R_v^{-1/2}h)^H (R_v^{-1/2}h)} = \frac{\alpha^2 |(R_v^{-1/2}s)^H (R_v^{-1/2}s)|^2}{\alpha^2 (R_v^{-1/2}s)^H (R_v^{-1/2}s)} = \frac{|s^H R_v^{-1}s|^2}{s^H R_v^{-1}s} = s^H R_v^{-1}s \quad (62)$$

Thus, the optimal matched filter is:

$$h = \alpha R_v^{-1}s \quad (63)$$

The expected value of the power of the filter output due to the noise to unity it is often normalized. That is:

$$E\{|y_v|^2\} = 1 \quad (64)$$

This constraint implies a value of α , for which it can be solved:

$$E\{|y_v|^2\} = \alpha^2 s^H R_v^{-1}s = 1 \quad (65)$$

Yielding

$$\alpha = \frac{1}{\sqrt{s^H R_v^{-1}s}} \quad (66)$$

Resulting on the normalized filter:

$$\alpha = \frac{1}{\sqrt{s^H R_v^{-1}s}} R_v^{-1}s \quad (67)$$

2.3 Statistical Data Analysis

2.3.1 Probability Distributions

Probability distributions are a fundamental concept in statistics. They are used both on a theoretical level and a practical level. The most important practical uses of probability distributions are:

- To calculate confidence intervals for parameters and to calculate critical regions for hypothesis tests.

- For univariate data, it is often useful to determine a reasonable distributional model for the data.
- Statistical intervals and hypothesis tests are often based on specific distributional assumptions. Before computing an interval or test based on a distributional assumption, we need to verify that the assumption is justified for the given data set. In this case, the distribution does not need to be the best-fitting distribution for the data, but an adequate enough model so that the statistical technique yields valid conclusions.
- Simulation studies with random numbers generated from using a specific probability distribution are often needed.

As has been stated previously, the mathematical definition of a discrete probability function, $p(x)$, is a function that satisfies the following properties:

1. The probability that x can take a specific value is $p(x)$. That is $P[X = x] = p(x) = p_x$
2. $p(x)$ is non-negative for all real x .
3. The sum of $p(x)$ over all possible values of x is 1, that is $\sum_j p_j = 1$

where j represents all possible values that x can have and p_j is the probability at x_j .

One consequence of properties 2 and 3 is that $0 \leq p(x) \leq 1$.

Detailed information on a few of the most common distributions is available below. There are a large number of distributions used in statistical applications. It is beyond the scope of this thesis to discuss more than a few of these.

2.3.1.1 Normal Distribution

The general formula for the probability density function of the normal distribution is

$$f(x) = \frac{e^{-(x-\mu)^2 / (2\sigma^2)}}{\sigma\sqrt{2\pi}} \quad (68)$$

where μ is the mean and σ is the standard deviation parameter. The case where $\mu = 0$ and $\sigma = 1$ is called the *standard normal distribution*. The equation for the standard normal distribution is

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad (69)$$

The following figure shows several normal probability density functions.

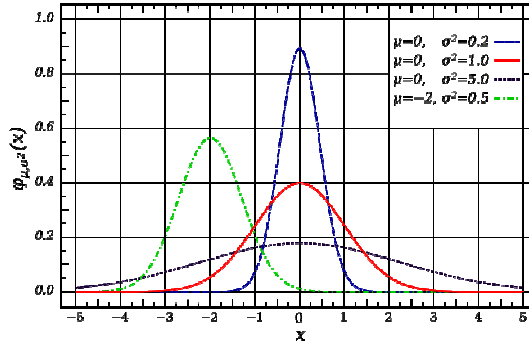


Figure 2.9: Normal Probability Density Functions. The red line represents the standard case.

2.3.1.2 Uniform Distribution

The general formula for the probability density function of the uniform distribution is

$$f(x) = \frac{1}{b-a} \quad \text{for } a \leq x \leq b \quad (70)$$

where a represents the location parameter and $(b - a)$ represents the scale parameter.

The case where $a = 0$ and $b = 1$ is called the *standard uniform distribution*. The equation for the standard uniform distribution is

$$f(x) = 1 \quad \text{for } 0 \leq x \leq 1 \quad (71)$$

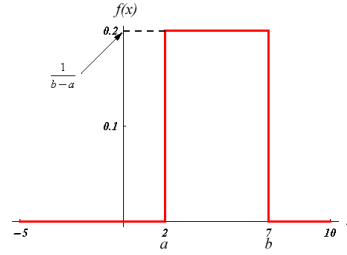


Figure 2.10: Uniform Distribution, in this case the parameters $a=2$ and $b=7$

2.3.1.3 Lognormal Distribution

A variable X is lognormally distributed if $Y = \ln(X)$ is normally distributed with " $\ln$ " denoting the natural logarithm. The general formula for the probability density function of the lognormal distribution is

$$f(x) = \frac{e^{-((\ln((x-\theta)/\mu))^2 / 2\sigma^2)}}{(x-\theta)\sigma\sqrt{2\pi}} \quad x \geq \theta; \quad \mu, \sigma > 0 \quad (72)$$

where σ is the standard deviation, θ is the location parameter and μ is the mean

The case where $\theta = 0$ and $\mu = 1$ is called the **standard lognormal distribution**. The case where θ equals zero is called the 2-parameter lognormal distribution.

The equation for the standard lognormal distribution is

$$f(x) = \frac{e^{-(\ln((x))^2 / 2\sigma^2)}}{(x)\sigma\sqrt{2\pi}} \quad x \geq 0; \quad \mu, \sigma > 0 \quad (73)$$

Next, several cases of log-normal distribution are shown on Figure 2.11

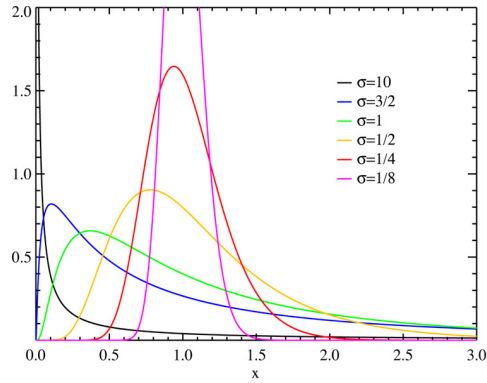


Figure 2.11: Log-normal distributions

2.3.2 Histograms

A histogram is the most common graphical representation of quantitative data that have been summarized in a frequency distribution. The values of the quantitative variable are shown on the horizontal axis. A rectangle is drawn above each class such that the base of the rectangle is equal to the width of the class interval and its height is proportional to the number of data values in the class.

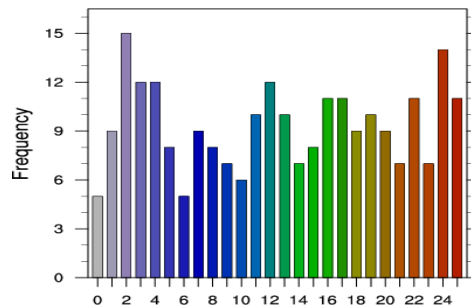


Figure 2.12: Histogram example

2.4 Nyquist Theorem: “Fundamental Theorem of DSP”

The Nyquist–Shannon sampling theorem is a fundamental result in the field of information theory, in particular telecommunications and signal processing, the last one been the area of study of this thesis.

Sampling is the process of converting an analogue signal into a numeric sequence (a function of discrete time or space). The theorem states that a discrete time sequence of a sampled continuous function $V(t_n = n \cdot T_s)$ contains enough information to reproduce the function $V=V(t)$ exactly, provided that the sampling rate ($f_s = 1/T_s$) is at least twice that of the highest frequency contained in the original signal $V(t)$:

$$V(t) = \sum_{n=-\infty}^{\infty} V[n] \cdot \frac{\sin[\pi f_s (t - nT_s)]}{\pi f_s (t - nT_s)} \quad (74)$$

where:

T_s	Sampling time interval
$f_s = 1/T_s$	Sampling frequency
$V(t)$	Value of signal (voltage) at arbitrary time t
$V[n] = V(nT_s)$	Value of signal at time $t = nT_s$

The election of a sampling rate different from the value established by the Nyquist theorem, can lead to a condition of either under or over sampling.

When the sampling rate is lower than or equal to the Nyquist rate, this condition is defined as *under sampling*: it is impossible to rebuild the original signal according to the sampling theorem.

An example where the reconstructed signal built from data sampled at the under sampled Nyquist rate is way off from the original signal is illustrated below.

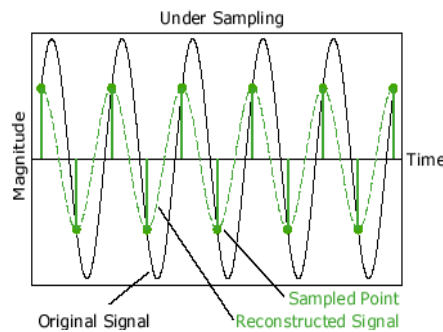


Figure 2.13: Reconstructed signal under the condition of under sampling

Under sampling causes frequency components that are higher than half of the sampling frequency to overlap with the lower frequency components. As a result, the higher frequency components roll into the reconstructed signal and cause distortion of the signal. This type of signal distortion is called aliasing. The Figure 2.14 reproduces the above aliasing argument in the frequency domain.

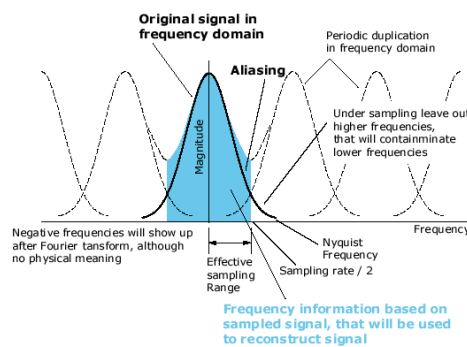


Figure 2.14: Spectrum of Sampled Signal at a rate lower than the Nyquist rate

Graphically, if the sampling rate is sufficiently high, i.e., greater than the Nyquist rate, there will be no overlapped frequency components in the frequency domain. A "cleaner" signal can be obtained to reconstruct the original signal. This argument is shown graphically in the frequency-domain schematic in Figure 2.15.

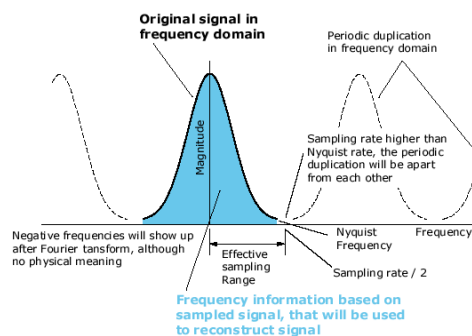


Figure 2.15: Spectrum of Sampled Signal at a rate higher than the Nyquist rate

CHAPTER 3

Experimental Work

The work carried out inside this thesis, at INRiM includes the study and analysis of the signal received from a superconducting TES working as single photon detector. In this context I have developed some off-line processing routines of the data collected on different measurement journeys at the “Cryogenic Innovative Devices” Laboratory at INRiM. This chapter aims to illustrate the scenario, and to describe the methodology used to acquire the data and process it. The results of these analyses will be presented along with some of the software routines developed to process the TES received signal.

3.1 Scenario

Nowadays, there is a variety of applications in quantum optics that need detectors with photon-number resolving capability, very low dark count rates and high quantum efficiency at visible and telecommunications wavelengths. In this sense, Transition-Edge Sensors are becoming fundamental for applications like astronomy [42,43] and quantum information [44] due to their intrinsic energy resolution and photon-number resolution. At the INRiM’s “Innovative Cryogenic Devices”

Laboratory, different Transition-Edge Sensors have been fabricated and characterized consisting of either TiPd or TiAu bilayers. TiPd TES are still under development. The data presented and analyzed in this thesis is referred to the first TiPd TES characterized as single photon detector.

For the realization of this work the first TiPd superconducting TES constructed at the laboratory was used in order to obtain the data that would be later processed

3.1.1 TES Single Photon Detector Characteristics

The device investigated in this thesis is a $20\text{ }\mu\text{m} \times 20\text{ }\mu\text{m}$ TESs, consisting of Ti/Pd/Ti multilayers sequentially evaporated by e-gun on $500\text{ }\mu\text{m}$ double side polished Si wafer covered on both sides by 500 nm thick SiN layer (grown by LPCVD).

The first $10\text{-}15\text{ nm}$ Ti layer was deposited on SiN substrate in order to improve the adhesion of the Pd layer (16 nm). The last Ti layer (65 nm) is the actual superconducting layer proximized by the Pd layer. The devices were fabricated by standard photolithography process and etched by RIE (Ti) and ion-milling (Pd).

The superconducting Al wiring, with thickness between 100 and 150 nm , was defined by lift-off technique combined with rf-sputtering of the superconducting films.

In Figure 3.1 the Resistance (R) vs Temperature (T) curve is shown. From the curve a transition temperature $T_c = 104.8\text{ mK}$, with its corresponding transition temperature interval $\Delta T_c = 0.5\text{ mK}$, and a normal resistance $R_N = 1.6\Omega$ can be obtained. The sensor resistance was measured using four wire readout and lock-in amplifier with typical sinusoidal excitation current of $0.2\text{ }\mu\text{A}$ at 37 Hz .

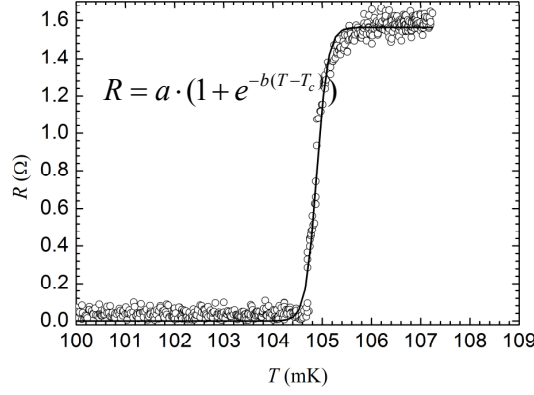


Figure 3.1. Resistance R vs Temperature T curve of a Pd/Ti TES at a excitation current $I = 0.2\mu\text{A}$. In the equation that describes the resistance during the transition a and b are constants, in this case $a=1.56\Omega$, $k=8.96 \text{ mK}^{-1}$ and $T_c=104.8 \text{ mK}$.

To characterize the device from a thermal and electrical point of view, is necessary to considered the relaxation time necessary to the sensor after an incident photon has been absorbed. This time constant is the thermal time $\tau_{th} = C/G$ where $G(T) = dP/dT$ is the total thermal conductance of the detector. It is derived from the power $P(T) = kV(T^n - T_{bath}^n)$ that flows from the TES at temperature T to the thermal bath at temperature T_{bath} , obtaining $G(T) = nkVT^{n-1}$ where n is the thermal coupling power-law exponent (with value from 4 to 5), V is the TES volume and k is the material dependent constant. If we introduce the average thermal conductance $G_{av}(T) = P/(T - T_b)$ [45] and we substitute P and express G_{av} in terms of $G(T)$, we can write

$$G_{av}(T) = \frac{P(T)}{T - T_b} = \Sigma V \frac{T^n - T_b^n}{T - T_b} = \frac{G(T)}{nT^{n-1}} \frac{T^n - T_b^n}{T - T_b} \quad (75)$$

from which the thermal conductance of the TES at the transition temperature T_c can be deduced

$$G(T_c) = \frac{nPT_c^{n-1}}{T_c^n - T_b^n} \quad (76)$$

The TES is voltage biased with the circuit shown in Figure 1.3, where $R_{bias} = 9.5 \text{ m}\Omega$ in order to obtain the Electro-Thermal Feedback ETF[15]. The bias curves at $T_b=55\text{mK}$, which represents the trend of the TES current I_{tes} vs the bias current I_{bias} , are shown in Figure 3.2. The I_{tes} , which flows through the TES, is measured by a DC-SQUID used as a current-voltage amplifier thanks to the input coil L_{in} placed in series with the TES. Known the SQUID gain $A = 84212 \text{ V/A}$, it is possible to evaluate the corresponding current. From Figure 3.2, the superconducting branch can be distinguish for low values of I_{bias} , except for a few $\text{m}\Omega$ of parasitic resistance (line 1 in Figure 3.2), while for $I_{bias} > 65 \mu\text{A}$, the TES reaches its normal value of 1.6Ω . The intermediate region corresponds to the superconducting to normal transition where the TES operating point has been set.

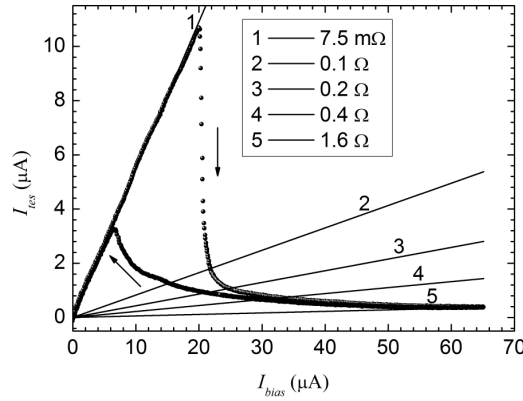


Figure 3.2 TES current (I_{tes}) vs bias current (I_{bias}) at 55 mK (dot curve). Solid lines represent the current distribution for fixed resistance values (as reported in the legend) of TES branch

3.2 Methodology

The methodology used in this work can be divided in three main areas, as follows:

- a. *Acquisition and characterization of the data:* before any analysis is performed it is necessary to characterize the data acquired by applying known statistical models.

- b. *Simplified Pulse Height Analysis*: a routine will be proposed to accomplish a satisfactory photon detection when the light emitting source is not triggered, discharging unwanted events, such as pulse pile-ups⁶
- c. *Filtering of the acquired signal*: consists on implementing different types of filters to improve the quality of the signals.
- d. *Evaluation and comparison of the Energy Resolution*: the computation of the histograms of the amplitudes of the detected pulses when both using the triggered and the non-triggered source, allows to evaluate the new energy resolution obtained when processing the signal with the DSP algorithm proposed in this thesis and consent to establish a comparison with the energy resolution of the non-processed signals.

3.3 Experimental Setup

Since the TES devices have a transition temperature ranging from 80 to 350 mK and the devices should work using strong electrothermal feedback, i.e. with a bath temperature lower than T_c a refrigeration system capable of reaching temperatures down to 50 mK is required. The cryostat used to do the measures from where the data for this thesis was obtained is an Air Liquide MINIDIL OD70-35mK ^3He - ^4He dilution refrigerator with a nominal cooling power at 100 mK of $\sim 25 \mu\text{W}$. In Figure 3.3 the dilution refrigerator used in our experiments is shown, in which it is possible to see the *4K stage*, where the SQUID is mounted, *the still at 800 mK stage*, the heat exchanger and the mixing chamber at the base temperature of 35 mK, where the sample holder is mounted.

⁶ In photon-detection, the dead-time is defined as the time after the arrival of a photon the system is busy. If N photons are arriving at the detector within a time-interval shorter than the dead time, a pile-up of photons may be the result. For a single-photon-counting detector this means that the N photons are counted as one and consequently a non-linearity between the actual photon intensity and the registered count rate is observed

A dilution refrigerator is a cryogenic device first proposed by Heinz London. When a mixture of the two stable isotopes of helium is cooled below a critical temperature, it separates into two phases. The lighter “concentrated phase” is rich in ^3He , and the heavier “dilute phase” is rich in ^4He [41]. The concentration of ^3He in each phase is temperature-dependent. Since the enthalpy of the ^3He in the two phases is different, the “evaporation” of ^3He from the concentrated phase into the dilute phase may provide highly effective cooling. In a gross simplification, the concentrated phase of the mixture is pretty much liquid ^3He , and the dilute phase is effectively ^3He gas. The ^4He composing the bulk of the dilute phase is inert and non interacting, and may be neglected. The evaporation of ^3He from the “liquid” phase to the “gas” phase cools the sample holder. This process works even at the lowest temperatures because the equilibrium concentration of ^3He in the dilute phase is finite even at zero temperature.



Figure 3.3. Dilution refrigerator composed by the 4K stage, where the SQUID is mounted, the still at 800 mK, the heat exchanger and the mixing chamber.

At the beginning of the operation, the 4K flange is used to condense the ^3He - ^4He mixture in the dilution unit. It does not cool the mixture sufficiently to form the phase boundary, but simply to bring it to 1.2K. Phase separation may be attained only once the temperature falls below the tri-critical point at 0.86K. This cooling is provided by the still; incoming ^3He is cooled by the still before it enters the heat exchangers and mixing chamber. Gradually, the rest of the dilution unit cools to the point where phase separation occurs. It is important that the ^3He concentration and volume of the mixture are chosen correctly, so that the phase boundary occurs inside the mixing chamber and the liquid surface lies in the still 4K. If this is not done, the fridge will not cool to base temperature. Thus it is CRITICAL to preserve the balance of the mixture.



Figure 3.4. The entire experimental setup composed by the dilution refrigerator inside the dewar with the Gas Handling System (GHS) for the injection of the ^3He - ^4He mixture.

During continuous operation, the ^3He must be extracted from the dilute phase (to prevent saturation) and resupplied to the concentrated phase. The ^3He is pumped away from the liquid surface in the still, where at $\sim 0.8\text{K}$, ^3He evaporates preferentially (1000 times faster than ^4He). ^3He leaving the mixing chamber is used to cool the returning flow of concentrated ^3He in a series of heat exchangers. A room temperature vacuum pumping system is used to remove ^3He from the still and compress it before passing it through impurity-removing filters and 77 K cold trap and returning it to the cryostat. The inflowing mixture is pre-cooled by the main helium bath and condensed on the 4K pot. A flow impedance (in the form of a capillary tube) is used to maintain a sufficiently high pressure in the 4K pot region for the gas to condense.

The picture in Figure 3.4 shows the dilution refrigerator inside the dewar connected with the Gas Handling System (GHS) for the injection of the mixture and the instruments used for the measurements. The experimental apparatus is mounted on the mixing chamber to ensure adequate thermal contact. The sample holder has been made with OFHC copper to maximize the material conductivity and it is shown on the left of Figure 3.5. On the right side of this figure the packaging designed for the TES can be seen. The device has been placed upon a little OFHC copper sideboard and with Al wire bonding it was joined to the connector. This connector is used to connect both the R_{bias} , made by 100 μm thick copper wire and the SQUID, placed on the 4 K stage, by means of two 69 μm thick NbTi wire, to the TES. The sideboard can be placed both on the vertical or horizontal wall of the sample holder using brass screws.

On the three stages of the dilution, still-4K and mixing chamber, there are three thermometers to check the temperatures of the cryostat during its function in order to measure the transition temperature and to control the bias point of the sample. To have a better control on the device, a thermometer has been placed on the sample holder as can be seen in Figure 3.5. The resistance of the thermometers are read out by an AVS 47 resistance bridge. The resistance bridge reads out the resistance through a four wire measurement: it applies a small current to the resistive

thermometer through two wires, and it measures the potential difference via the other two. The sensitivity of the temperature measurements is limited by electronic noise in the thermometer.

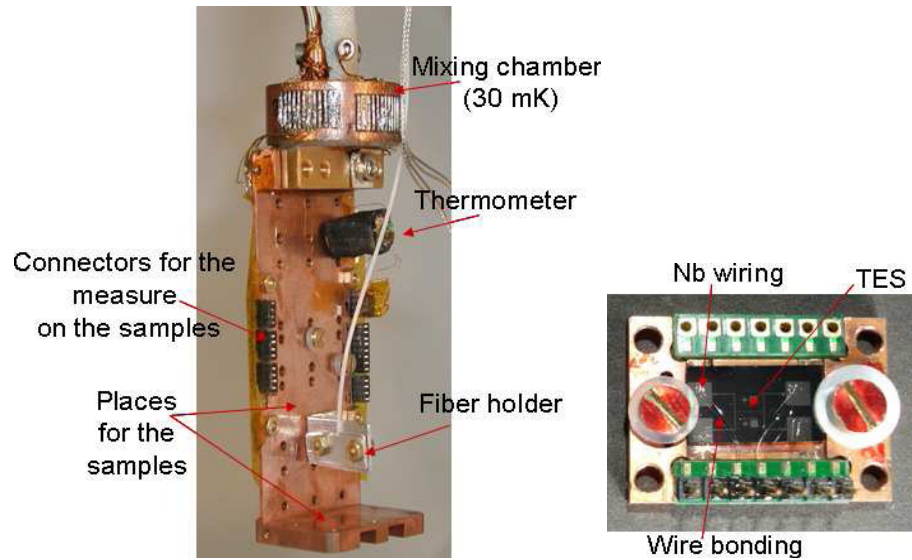


Figure 3.5. On the left the sample holder made with OFHC copper can be distinguished. On the right the packaging created for the TES is shown.

CHAPTER 4

Digital Signal Processing

4.1 Data Acquisition

The data acquisition process is performed by using a LeCroy oscilloscope model WR44xi that allows storing the data in *ascii* format.

The processing of the acquired data is accomplished with the use of the Matlab® Engineering Software, which is a high-level language and interactive environment that allows performing computationally intensive tasks.

4.1.1 Noise Modelling

The sampling time interval chosen to discretize the noise signal corresponds to a distance of 100 ns between samples; according to the Nyquist-Shannon Theorem explained in Chapter 2, this is consistent with an oversampling of the signal. This condition has been set intentionally to obtain the highest possible resolution of the reconstructed signal, for its further characterization and analysis, in both time and frequency domains.

Since it is crucial describing the noise present during the experimental measurements through a known statistical model, several data frames⁷ were acquired with the light emitting source of the circuit turned off.

A routine in Matlab® (see Appendix A) has been developed to open the data files from the oscilloscope. Once the data was read and stored in the computer, some analyses were made in order to determine if the noise could be represented following the white Gaussian noise model.

Regarding the time analysis made, the noise sample mean and sample variance were calculated with Matlab® computational tools. Examples of noise data frames from different measurement journeys in which the mean and the variance can be distinguished are presented in Figure 4.1 and Figure 4.2.

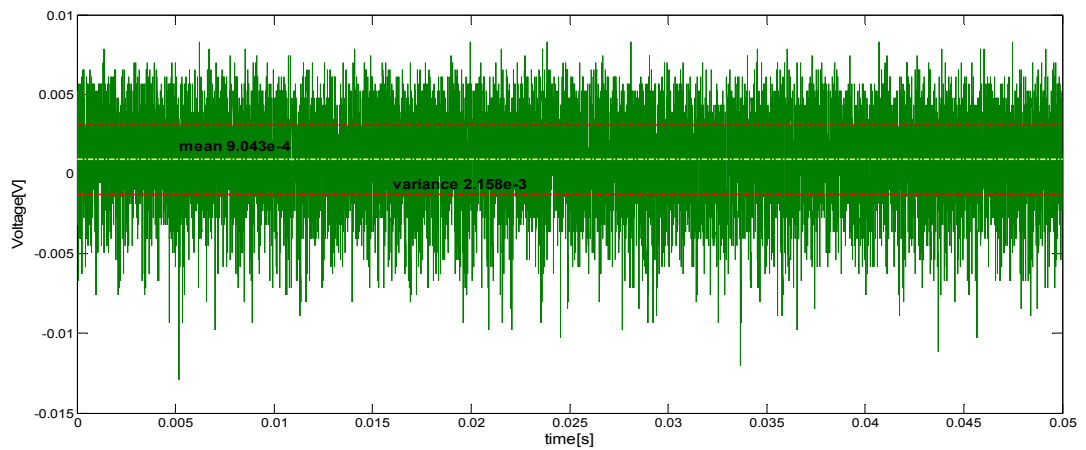


Figure 4.1: Generic Noise Data Frame (I)

⁷ A data frame refers to a data structure containing the variables acquired in this case with the use of an oscilloscope

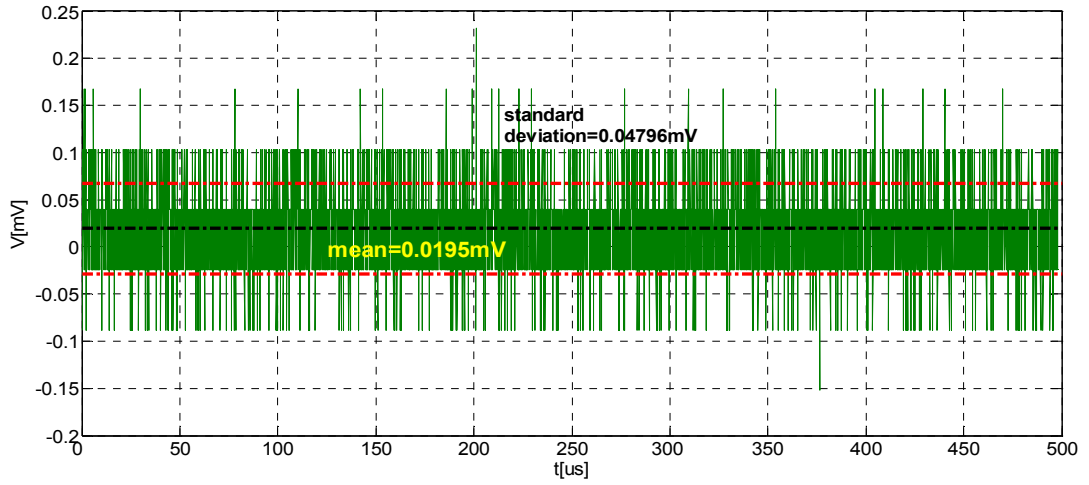


Figure 4.2. Generic Noise Data Frame (II)

On a first approach, it is likely to assume that the noise follows a random pattern with a non-zero mean, which could be characterized by the white Gaussian model.

However, when zooming in the case of *Noise Data Frame I* from Figure 4.1, a more detailed view shows that the noise signal has a very particular behaviour; it can be notice a persistent oscillating component, which in some cases could be considered as the possible arrival of a photon. For instance, note the behaviour of the signal around the time instants $t_1=0.7 \cdot 10^{-4}$ s and $t_2=2.55 \cdot 10^{-4}$ s; the signal at these two points seems to have a fast leading edge after which there is a “tail” of what could be a photon-generated pulse.

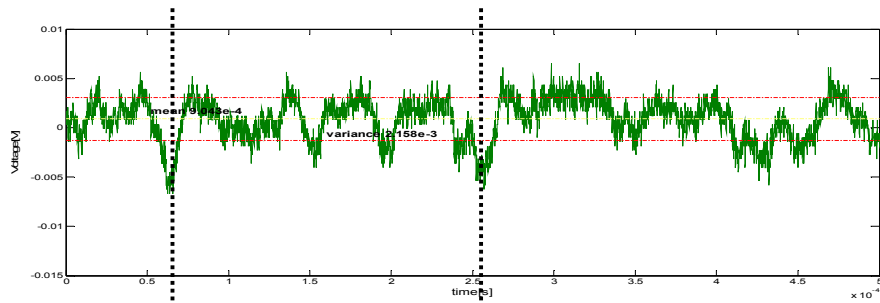


Figure 4.3. Zoomed-in Noise Data Frame I

As concerns the frequency, two analyses were performed on the signals acquired. First, spectral estimation was performed via periodogram⁸, the corresponding Matlab® routine developed to carry out this operation can be found on the Appendix. Regarding the collection *Noise Data Frames I*, the most significant noise components in both low and high frequencies are shown in the figures below.

⁸ Periodogram is a nonparametric spectral estimation method, in which the PDS is estimated directly from the signal itself. Consists on calculating the discrete-time Fourier transform of the samples of the process (usually done on a grid with an FFT) and take the magnitude squared of the result. See section 2.2.3 (applications of the DFT).

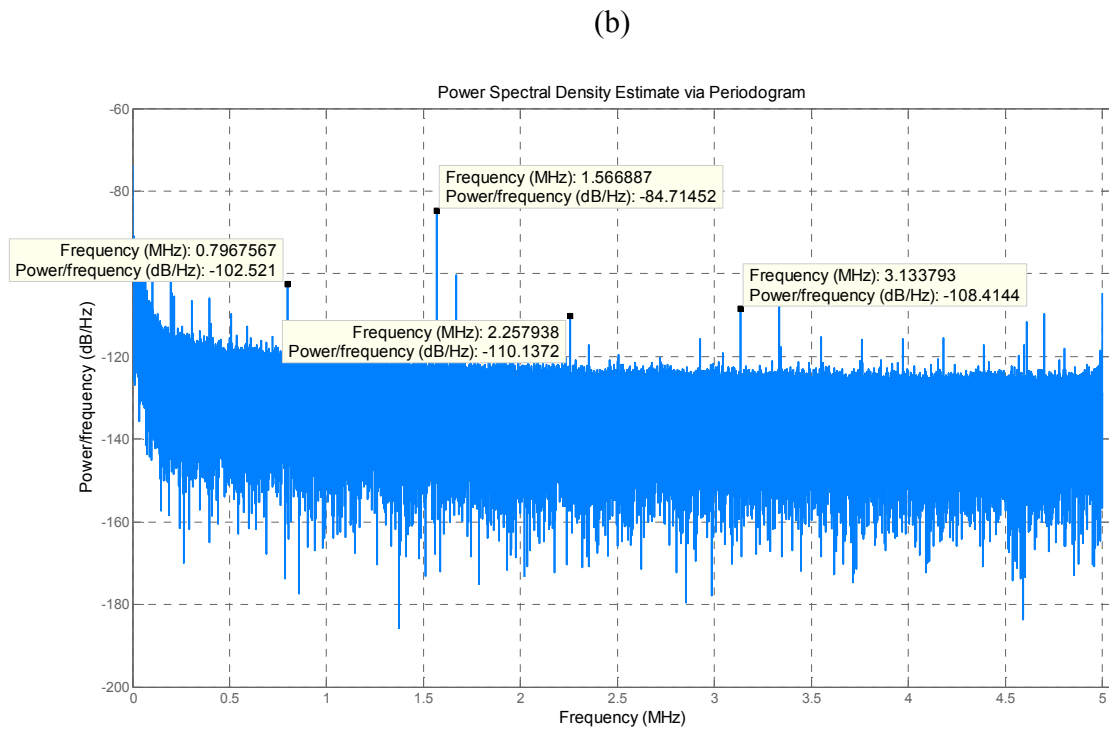
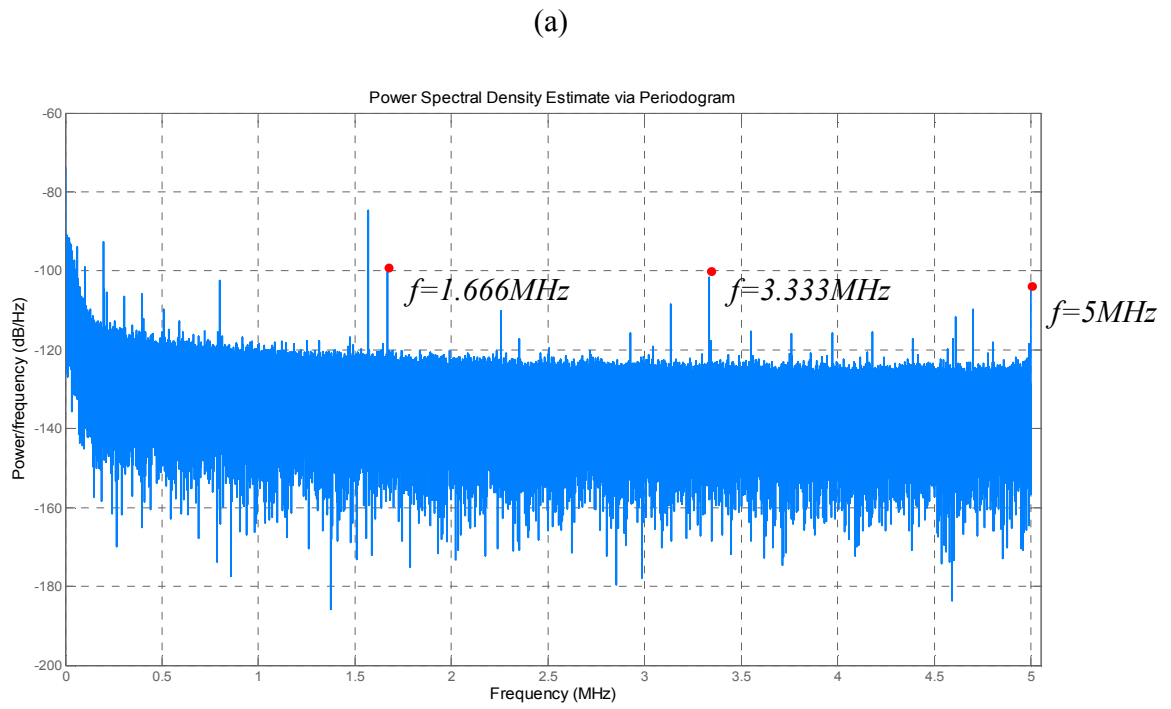


Figure 4.4: Noise Spectral Estimation via Periodogram, corresponding to Noise Data Frames I.

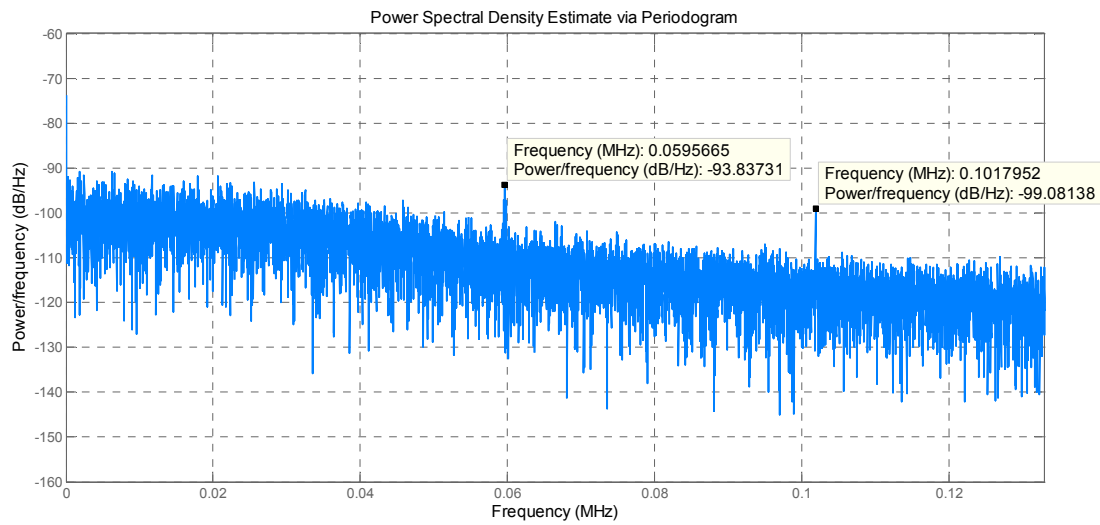


Figure 4.5: Low-frequency Noise contributions and harmonics

Once gathered the data from this measurement journey, the sample noise distribution function was calculated, with the help of the Distribution Fitting Toolbox of Matlab®. The results obtained are shown in Figure 4.6.

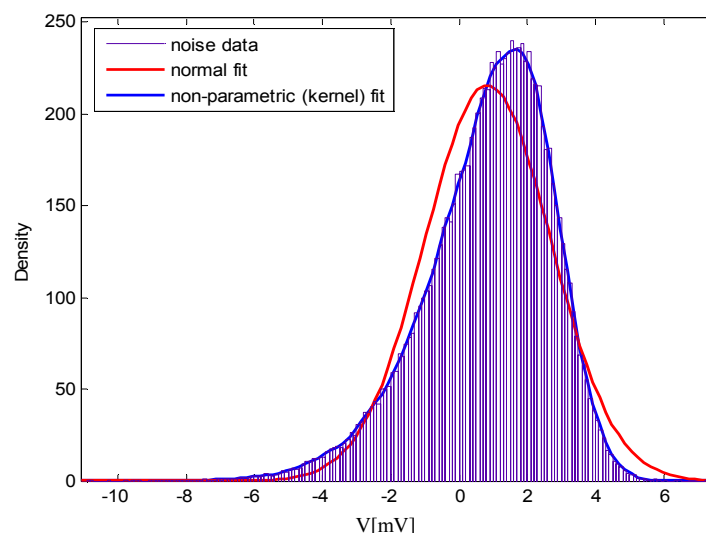


Figure 4.6: Noise Sample Probability Density Function

It can be seen how in this case the noise present on the experimental measurements does not follow a behaviour determined by the normal or Gaussian distribution. The

non-parametric fitting⁹ is clearly non symmetric and has a non-zero mean. This affects directly the digital signal processing, because for all the processing operations it was assumed a White Gaussian noise model. Assuming different noise models goes beyond the limits of this work and would require the formulation of advanced mathematical models and more complex algorithms.

However, regarding the *Noise Data Frames II*, the results obtained were quite different. In this case, the noise had a behaviour more similar to the white Gaussian model that is assumed in this thesis.

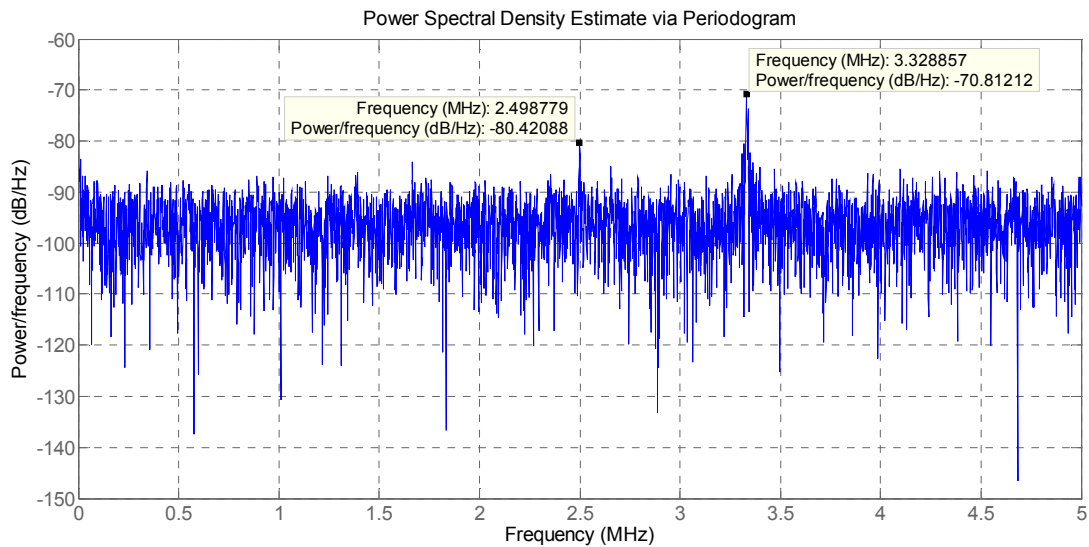


Figure 4.7. Power Spectral Density corresponding to Noise Data Frames II

In Figure 4.7 can be noticed that the noise has a relatively uniform distribution along the studied frequencies, consistent behaviour with the white Gaussian model. Two particular contributions can be distinguished around the frequencies of 2.5Mhz and 3.3Mhz, that are also present in the power spectral density of the *Noise Data Frames II*. Because of their manifestation in the different measurement journeys, it is probable that these noise contributions are been introduced by one of the components that constitutes the experimental circuit.

⁹ Smoothed curve that fits the data without extracting any parameters from the model

4.1.2 Signal Acquisition and Characterization

To acquire and characterize the signal obtained from the Superconducting TES single photon detector, the same Matlab® codes previously mentioned for the acquisition and preliminary analysis of the noise signal were used.

To generate the optical signal, two different optical sources were employed: a triggered 690 nm laser diode source and a non triggered monochromator functioning at a wavelength of 200 nm.

In these two cases, the pulse shape of the detected signals are similar. However, the algorithms employed to process the data in each case are different because of the presence or absence of the trigger: it allows synchronizing the oscilloscope with the source, specifically with the occurrence of the photon emission.

In the non-triggered case, it is necessary to pre-process the data acquired. A Matlab® code routine is applied in order to determine when the arrivals of the photons occur. In this way, only the useful data sub-frames are considered for its later processing.

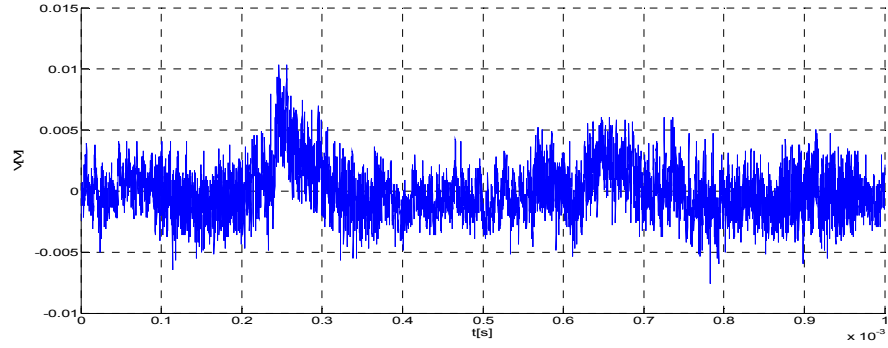
When using the laser instead, the emission of photons occur in a determinate instant established by the trigger, hence the oscilloscope is synchronized in a way that only the data frames corresponding to the arrivals of photons are acquired. In this case, the storage of the data is more efficient when comparing to the non-triggered case, because only the useful data is acquired.

A sampling time of 100ns was used to sample the signals in both cases.

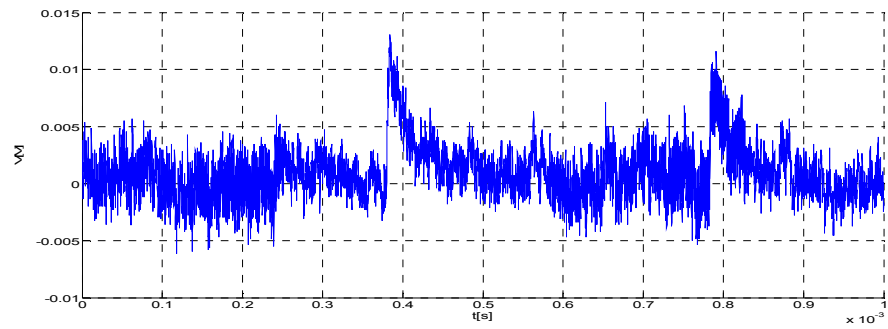
Next, several figures are presented where the waveforms of the signals acquired are shown.

Figure 4.8 corresponds to the acquired signal generated by the monochromatic source. It is worth to notice that not all of the data sets acquired, in this case (c), contain an event (the arrival of a photon); therefore the whole data frame acquired is useless, and does not contain any useful information for the analysis that will be performed later.

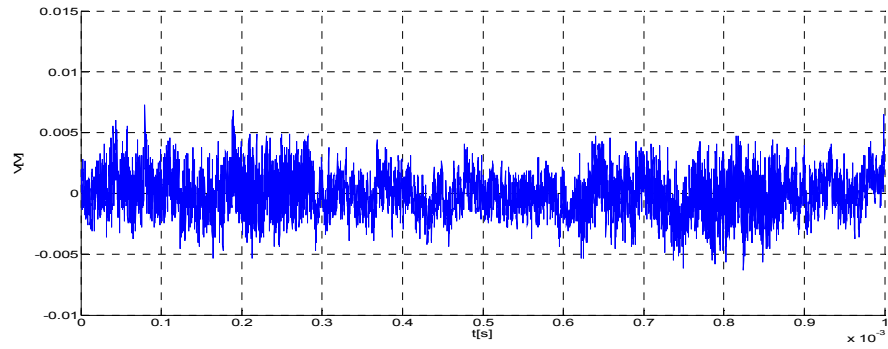
Figure 4.9 corresponds to the signal generated by the Laser (triggered source). It can be seen how in all the data frames the photon arrival event occurs at the very same time: the events are aligned between them. Not necessarily a photon or a group of them will always be detected by the TES device, the photodetection phenomenon is determined by a variety of aspects, such as the power of the optical source and the coupling between the elements of the circuit; however, if the photons are detected, the corresponding generated-pulse will be synchronized with the occurrence of the trigger.



(a)



(b)



(c)

Figure 4.8: Detected signals generated by the monochromatic source at 200 nm. Figures (a) (b) (c) show an example of the data frames acquired in this case, notice that the arrival of photons occurs randomly

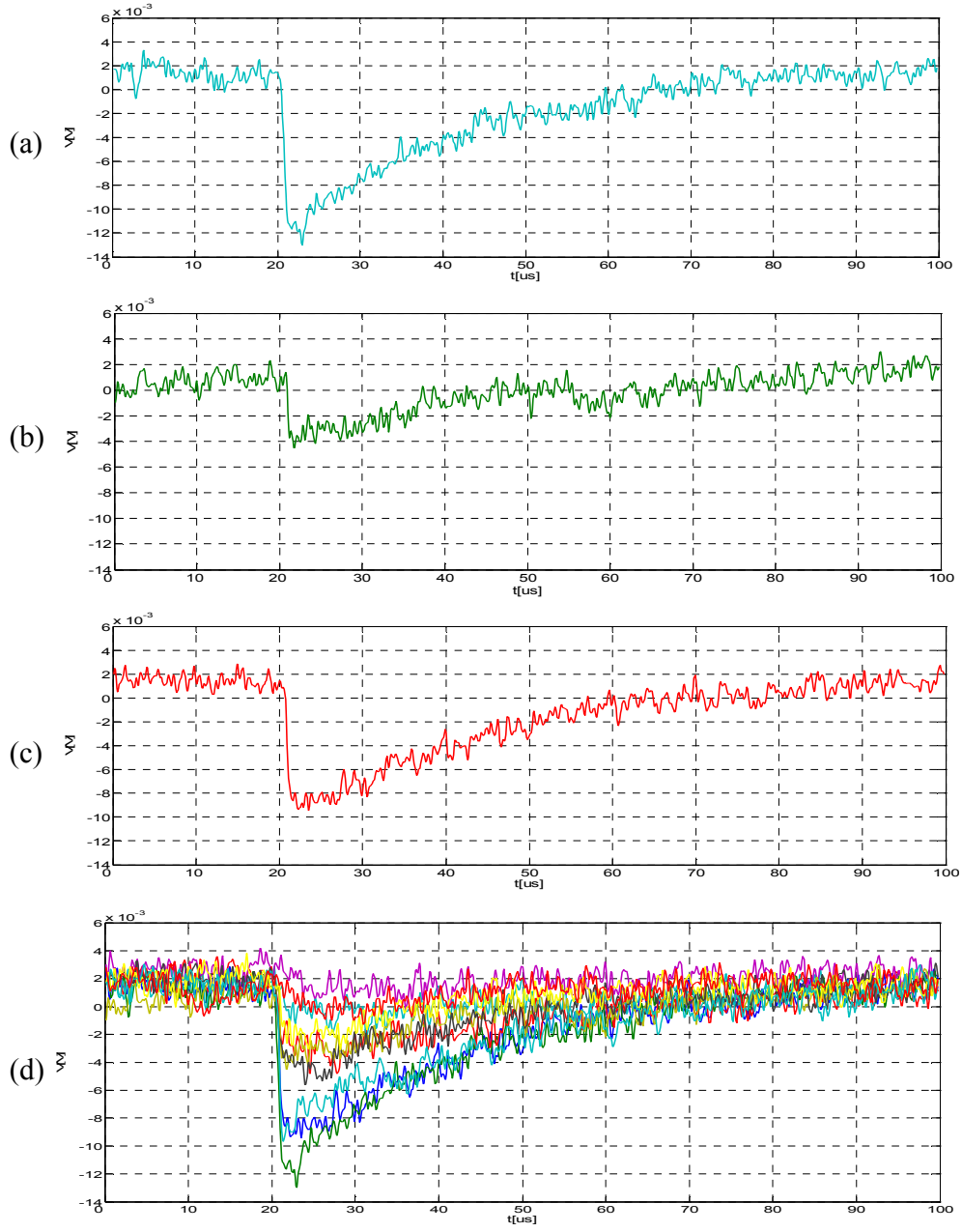


Figure 4.9: Detected signals generated by the Laser Source at 690 nm. Figures (a) (b) (c) show an example of single pulses. Figure (d) shows the plot of several impulses of different energies.

4.2 Pulse Height Analysis

4.2.1 Theoretical Basis

The pulse height analyzer (*PHA*) was originally developed to measure the amplitude of pulses from nuclear radiation detectors. It is therefore an important part of most systems that are used for analyzing event energies from alpha, beta, gamma, and X rays, for analyzing the time between events (from a time-to-pulse-height converter), and for ensuring orderly storage of count information in sequential time periods.

PHA's are ideally suitable in system configurations for experimental and production applications in nuclear, chemical, medical, and materials analyses. PHA's are an outgrowth of the single-channel analyzer, which counted amplified electrical signals of a given pulse height created by nuclear events occurring within a detector. The amplitude was accepted by a peak voltage within a "window" or channel defined by two discriminators, referred to as a differential discriminator.

The input signals to the PHA system must be optimally processed to achieve the best signal-to-noise ratio [46]. Pulseprocessing convert the burst of charge, produced when a radiation particle or photon is absorbed by the detector, to a shaped pulse (usually Gaussian). The quantity of charge produced is related to the energy of the incident radiation. A charge-sensitive preamplifier/voltage amplifier combination serves to amplify and shape the charge into a pulse with an amplitude (peak voltage) proportional to this charge, hence to the energy lost in the detector.

The pulses, which are random in both time and amplitude, are fed to an ADC and recorded in an address register and their number stored in a memory. When an event takes place, the ADC provides a channel address proportional to the input pulse amplitude and transfers the digitized peak voltage information to the channel address register. The count information in that channel is placed in the data register and updated by one count. The updated data is then placed in memory.

Figure 4.10 shows a simplified block diagram of a generic pulse height analyzer device.

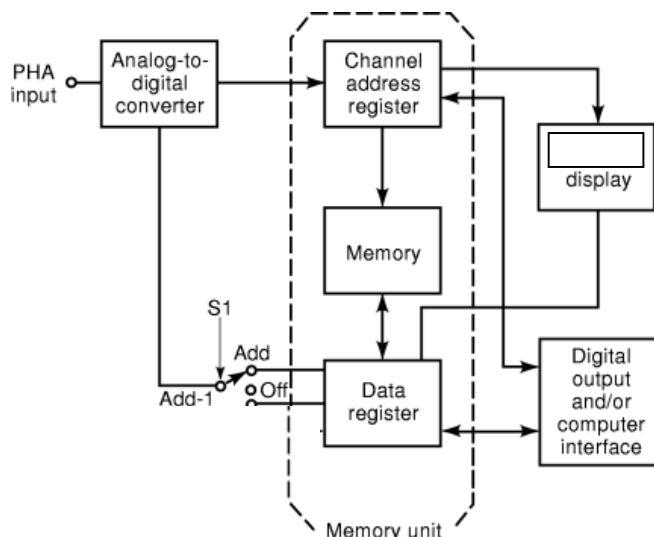


Figure 4.10: Simplified PHA block diagram illustrating the use of an analogue-to-digital converter to digitalize the analogue signal from PHA to generate the spectrum channel number. The spectrum is displayed or transferred to the computer.

This is the principle of operation of a pulse height analyzer. Systems that provide a variety of functions for the acquisition, storage, display, and interpretation of information from random events are referred to as multichannel analyzers (*MCAs*). The terms PHA and MCA are often used interchangeably; however, MCA is a broader term signifying a system that can be configured in a number of ways (including operation in the PHA mode).

The strong growth of the personal computer has had a major impact on the MCA. Many of the functions of a traditional MCA can be performed as well as or better by a PC. These include spectral display, data analysis, and control functions. Since some features of an MCA are unique, the best way to radically increase the performance of MCAs is by coupling the unique features of an MCA to a PC. In this work, the

functions usually performed by a MCA are emulated with a PHA routine developed using Matlab® Software.

4.2.2 Pulse Processing

The purpose of the pulse processing routine developed is analyzing the raw data from the detector in order to sense the occurrence of valid events and extracting the useful information contained in them such as the pulse height and the energy of each photon detected.

In the case of a non-triggered source, been necessary to determine the occurrence of an event, the DSP software routine watches the incoming data and detects the pulses that result when a photon hits the detector as explained before. When in a data frame an event is detected, the program collects the entire pulse and stores it along with its coordinates (temporal position and data frame number) for its later analysis. A pulse height analysis is then performed in order to get the best possible estimate of the height of each pulse. The importance of the pulse height stands on its proportionality to the energy of the detected photons, when the TES detector is working in its linear response region.

4.2.2.1 Pulse Detection

Since the objective of the pulse-detection code is to determine when a valid event occurs in the data stream, it is very important establishing a consistent definition of what is an event: an event is the consequence of a radiation interaction in the detector that leads to a recorded pulse, in the absence of undesired phenomenon (pile-up)

On a first approach, the goal of the pulse detection algorithm developed is to determine the occurrence of any event that could be considered a pulse generated by the arrival of a photon to the Superconducting TES photodetector. Once determined

these “possible events” the next step consists on analyzing them in order to establish if they are valid or not. Rising time, impulse width and the presence of pile-ups are taken in consideration to decide the validity of the pulses.

The algorithm proposed for the pulse detection is based on using a smoothed derivative of the data. The derivative is used in order to avoid the DC level of the data affecting the detection and to sense the rising fronts of the pulses. The derivative is calculated by convolving the data with a “box” derivative function. This convolution performs a smoothing on the data as well as finding the derivative of the original signal.

Figure 4.11, below, shows an example of the box derivative function. The number of elements of this “box” is an adjustable parameter that must meet two opposite requirements: on one hand it must be large enough for smoothing the noise component of the signal, while on the other hand it should be sufficiently small in order to be able to track fast variations in the input signals: leading edges of the pulses!

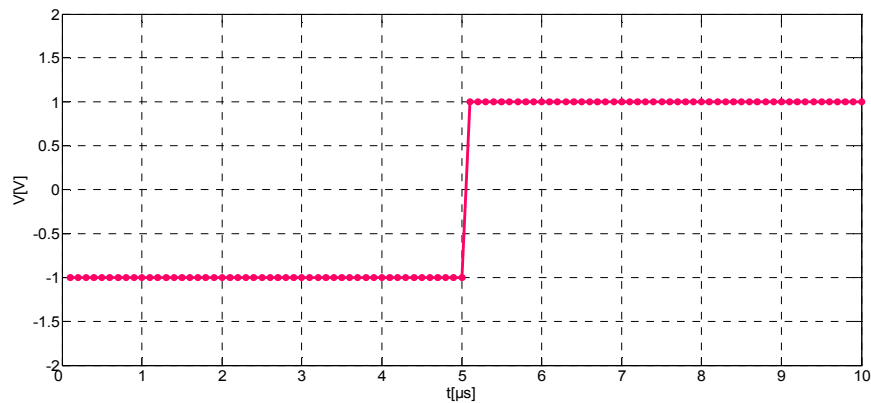


Figure 4.11. Demonstrative Box derivative function, similar to the one used to convolve the data in order to create a trigger for the detection of the pulses generated by the arrival of a photon.

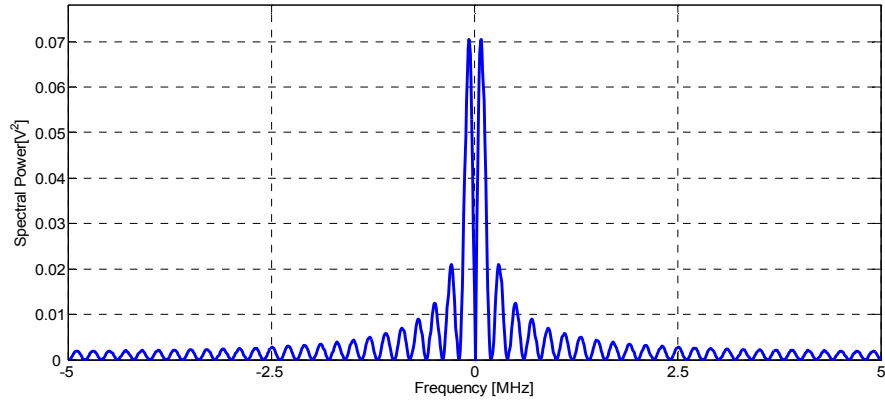


Figure 4.12. Fourier Transform of the Box derivative Function

In Figure 4.13 a first estimation of the pulses templates are presented when varying the length of the “box derivative function”. The duration of the data frame is equal to 1ms, therefore, the time dimension of the “boxes” considered were $10\mu s$, $50\mu s$ and $100\mu s$, that correspond to a quantity of 100, 500, and 1000 samples respectively. It can be seen how the leading edge of the pulse waveform is slowed down whilst the dimension of the box derivative function increases. Also, the smoothing performed on the signal, which in this case corresponds to a low-pass filtering, seems to worsen when using the larger “box”.

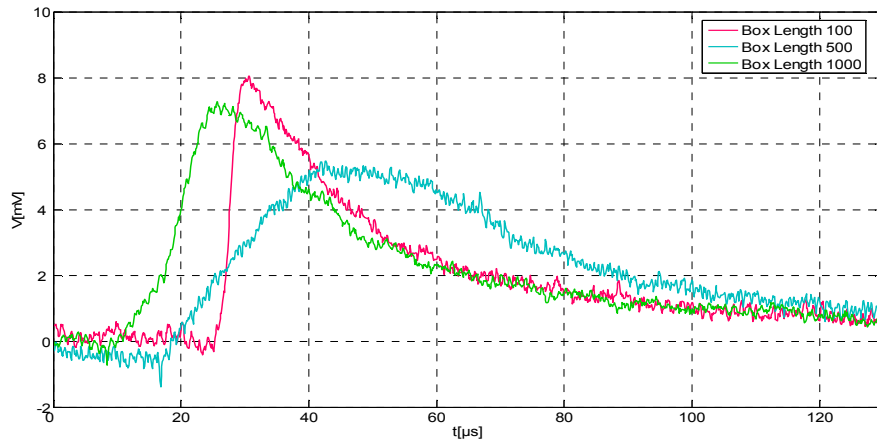


Figure 4.13. Pulse templates obtained when varying the size of the “box derivative function”

Figure 4.14 shows the derivatives of a signal data frame obtained with the use of the different “boxes” derivative functions. When the pulse-detection algorithm senses a pulse on the data stream, it simulates a trigger. Note that the best derivate (“generated-trigger”) is the one obtained when the “box derivative function” has a length of 100 samples, which in spite of having a larger high-frequency content reaches higher amplitude levels, and a better “trigger to noise” ratio in comparison with the other cases.

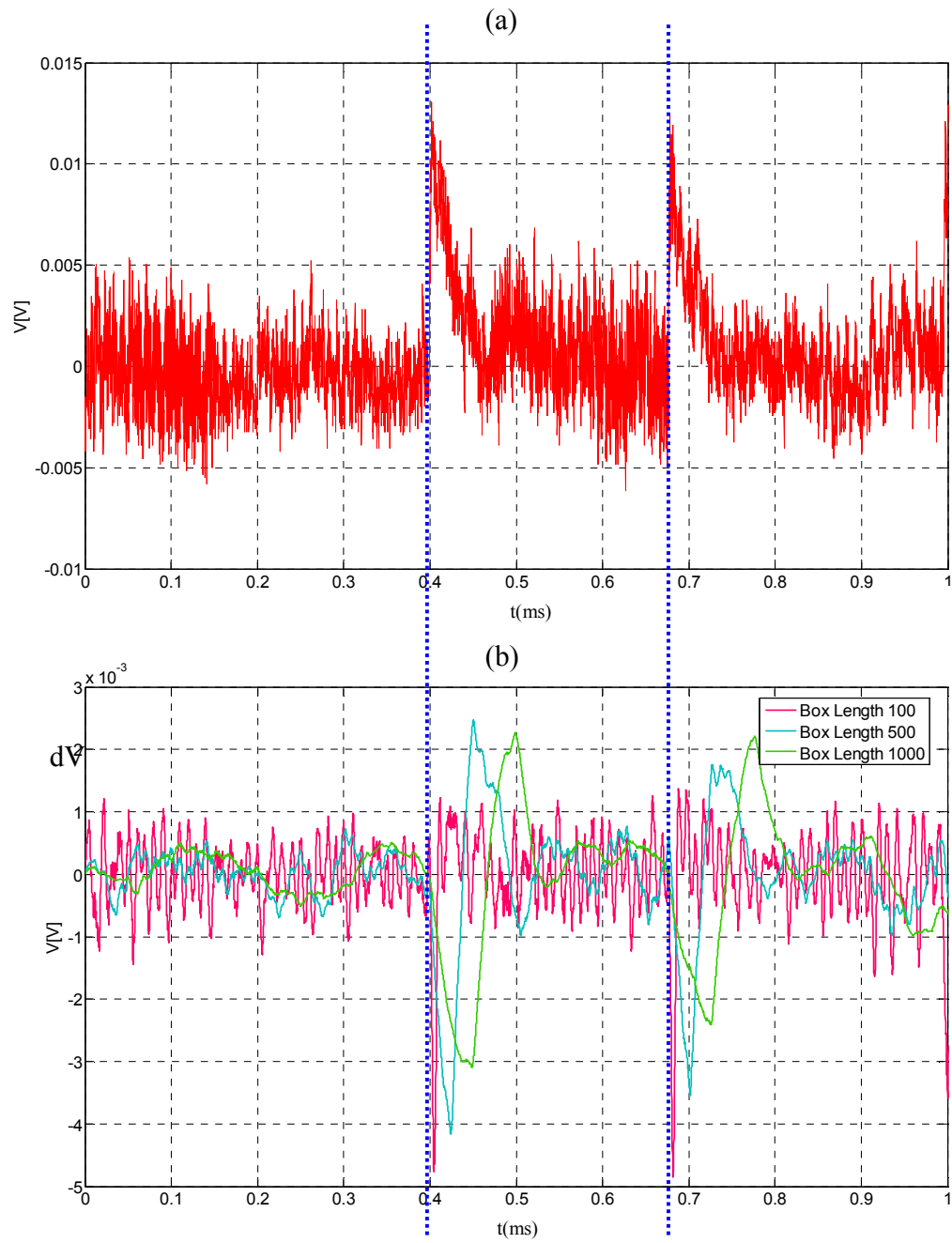


Figure 4.14. Signal derivatives obtained by applying the “box derivative function” to the original unprocessed signal from the monochromatic sources. (a) Signal from the monochromatic Source. (b) Derivative Functions using different “box” sizes

In Figure 4.15, an example of a generic impulse is presented, where two time instant and their amplitudes are specified. When examining several data frames, the estimated duration of the leading edge is $3\mu s$ while the decay time is around $0.1ms$. The best estimated representation for this impulse corresponds to the template obtained employing a box derivative function of 100 samples spaced in time $0.1ns$ between them.

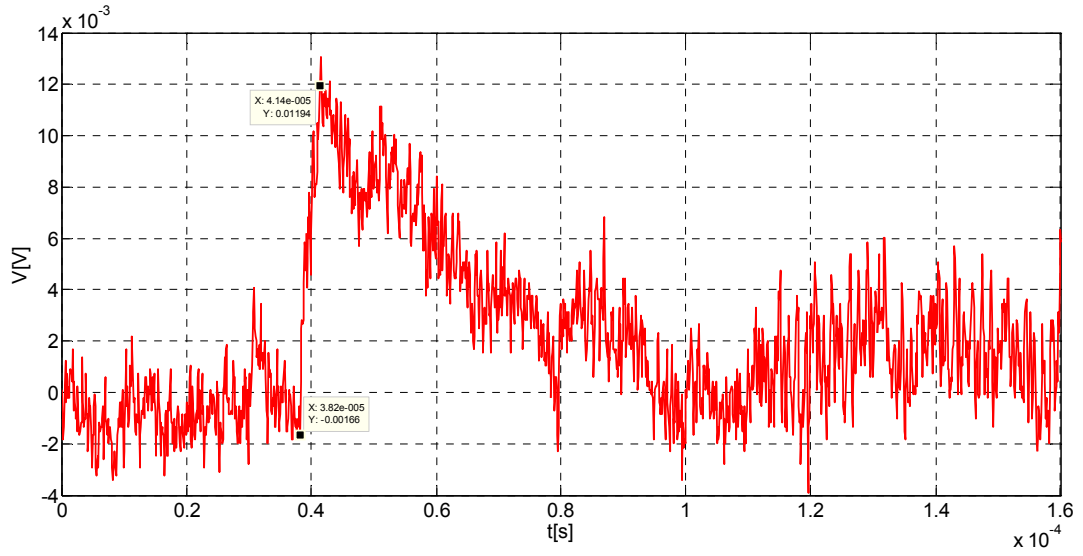


Figure 4.15. Non-processed generic impulse waveform from the non-triggered source.
The leading edge has a duration of $3.2\mu s$, while the decay time is around $0.1ms$.

Once the derivative of the data frames is obtained, the next step of the pulse-detection algorithm is fixing a threshold.

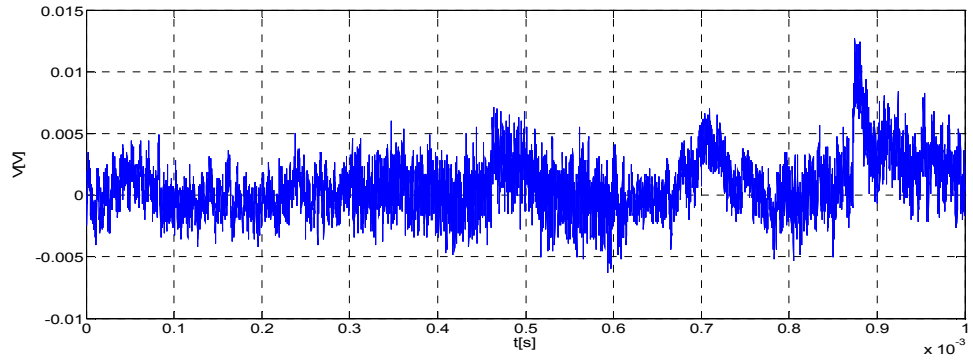
If the derivative of the original signal exceeds the fixed threshold the algorithm detects a pulse. Once again, this threshold is an adjustable parameter that must be set low enough in order that even the smallest pulses are detected, but high enough that the noise in the system would not produce false triggers.

Finally, when a pulse is detected, the processing algorithm begins counting down the length of a pulse data record while stores the corresponding data. The length of the pulse data record is the number of samples stored by the Pulse Height Analyzer for each event of photon detection.

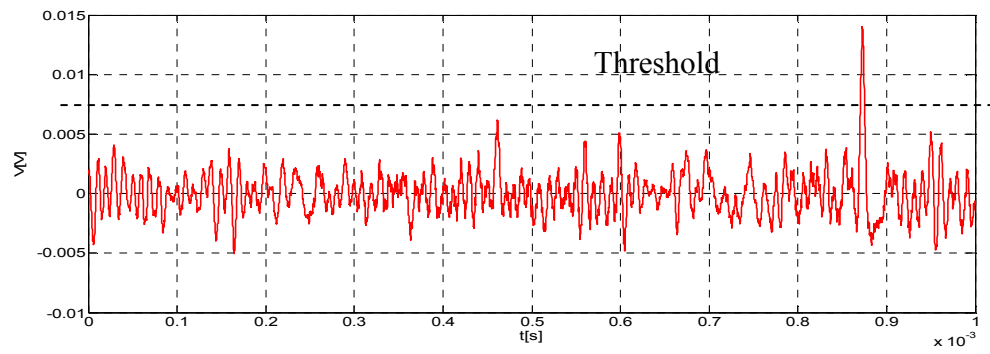
In the next pages some examples of the pulse-detection algorithm will be given. In Figure 4.16 a generic data frame is shown; in (a) a photon arrival can be distinguished, while in (b) the derivative of the signal is shown. Notice how the noise present in the data frame, especially around $t=0.5\text{ms}$ and $t=0.7\text{ms}$ is not recognized by the pulse detection algorithm as a valid event.

Another random data frame is showed in Figure 4.17, analogously to Figure 4.16, the photon arrivals events are shown in (a), in this case can be seen how the arrival of a photon occurs at the end of the frame and the derivative reaches values above the threshold established for the detection of a valid pulse. This is one of the many problems that presents when detecting a pulse. In this scenario the algorithm does not take this late pulse as valid due to the lack of signal data for filling the pulse data record in its totality.

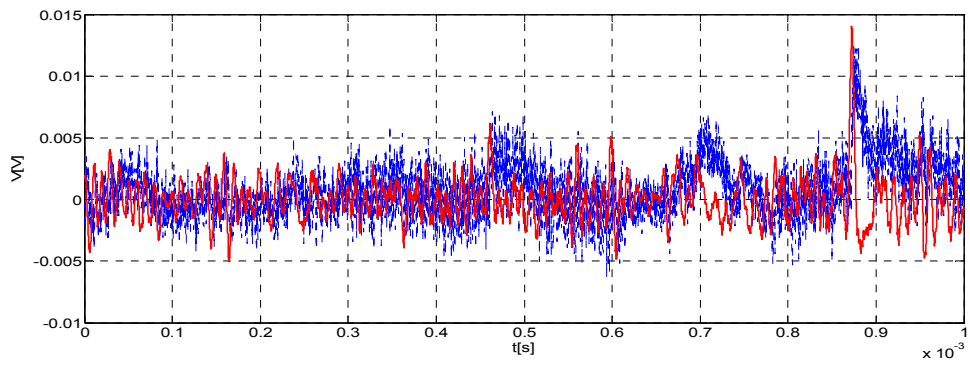
After the data frames have been analyzed and the pulse detection has been performed, the pulse records are stored into an array to continue with the next step of the DSP algorithm, where the presence of undesired phenomenon such as pile-ups is revealed. This phenomenon worsen the quality of the data for the subsequently analyses that needs to be performed, hence is necessary to develop a sub-routine to reject pile-ups. Before proposing a method for the elimination of pile-up events, some theoretical basis and concepts regarding Pile-up will be given on the next section.



(a)

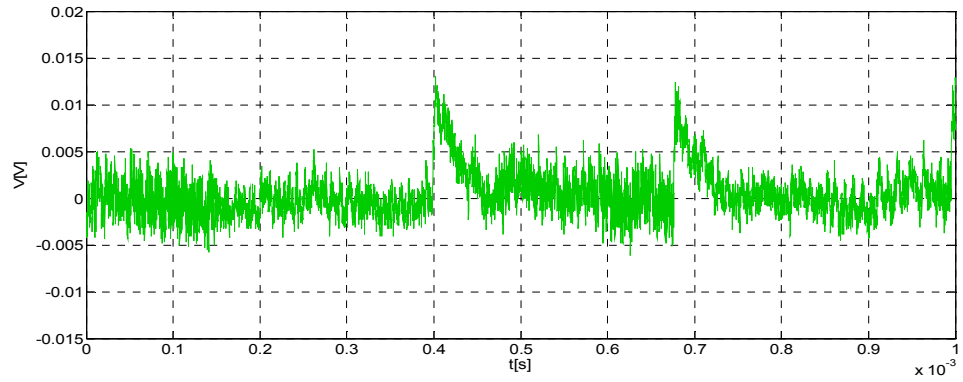


(b)

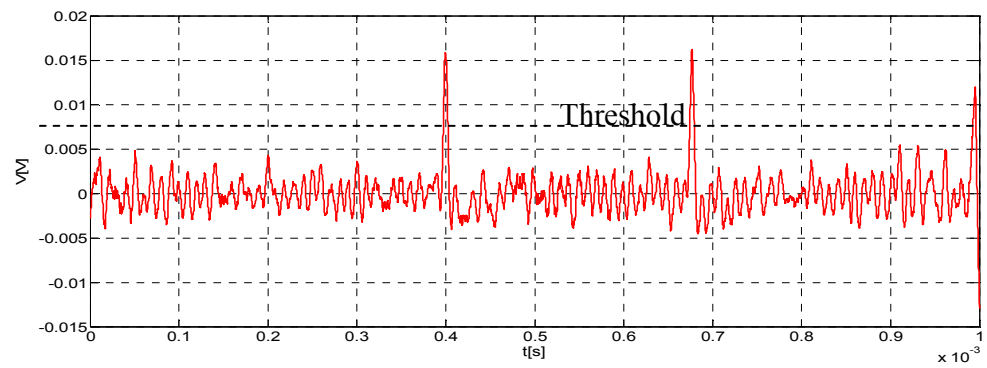


(c)

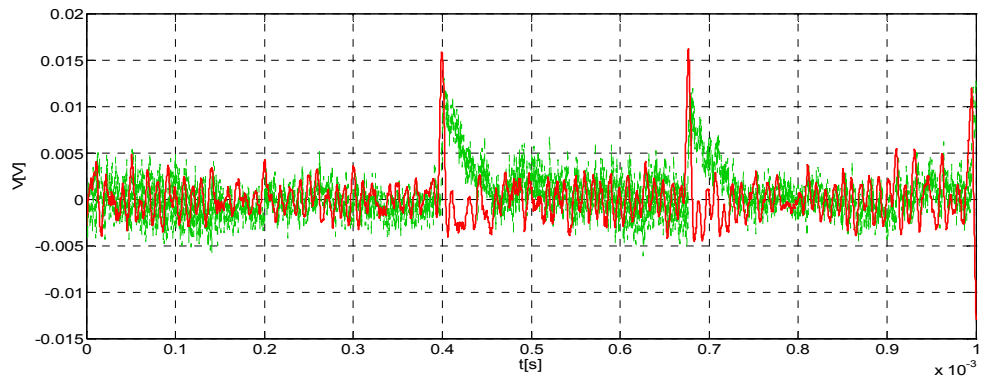
Figure 4.16. Pulse Detection I: (a) Signal data frame generated by a monochromatic source; (b) Derivative of the signal showed in (a); (c) Signal and derivative plotted simultaneously.



(a)



(b)



(c)

Figure 4.17. Pulse Detection II: (a) Signal data frame generated by a monochromatic source; (b) Derivative of the signal showed in (a); (c) Signal and derivative plotted simultaneously.

4.2.2.2 Pile-Up

The Phenomenon

The fact that pulses from a radiation detector are randomly spaced in time can lead to interfering effects between pulses when counting rates are not low. These effects are generally called *pile-up* and can be minimized by making the total width of the pulses as small as possible.

Pile-up phenomena are of two general types which have somewhat different effects on pulse height measurements. The *tail pile-up* involves the superposition of pulses on the long-duration tail (see Figure 4.18). Under conditions described earlier, tails can have durations of relatively long periods of time so that the pile-up can be significant even at relatively low counting rates.

The effect on the measurement is to worsen the resolution by adding wings to the recorded peaks in the pulse height spectra as shown in the figure. The remedy for tail pile-up is applying active baseline restoration techniques. Pulse shapes that return quickly to the baseline will also help eliminate the irreducible tail pile-up caused by the normal decay of the shaped pulse.

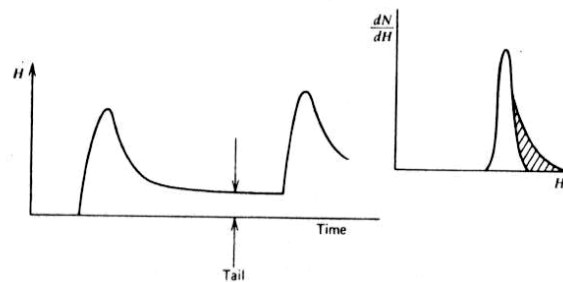


Figure 4.18. Pile-up from the tail of a preceding pulse. The effect on the differential pulse height spectrum for constant amplitude pulses is shown as the cross-hatched area at the right.

The seriousness of peak pile-up for a given situation can be estimated from the counting rate and the effective width of the signal pulses. The effective width is

difficult to define, except for rectangular pulses, but can be approximated by the FWHM¹⁰ of the first lobe of the shaped pulse.

Pile-up can also influence measurements other than the pulse height distribution. In particular, timing measurements can be affected by the resulting changes in pulse shape that accompany pile-up. The study of these effects goes beyond the scope of this thesis, for further information on the subject refs [47,48] describe the quantitative losses that can occur in coincidence experiments based on crossover timing when pile-up distorts the pulse shape; Wilkinson [49] also points out the errors that can occur in the time interval measurements when variable counting rates are encountered in the course of the measurement.

Nowadays, the availability of digital pulse-processing systems opens the possibility of doing more than simply tolerating the losses associated with pile-up.

If a detailed digital sampling is made of each output pulse shape from a pulse processing system, it should in principle be possible to distinguish those that originate from a single event from those involving pile-up in which more than one event have contributed.

The possibility then arises of deconvolving the piled-up pulses into their separate components through a detailed analysis of their shape.

The figure below shows an example of a pile-up pulse that actually arises from two events in the detector. Since the shaping process produces pulses of a predictable shape when operating on isolated single events, pile-up pulses depart from this simple profile and can therefore be selected for further analysis.

¹⁰ Full Width at Maximum Height

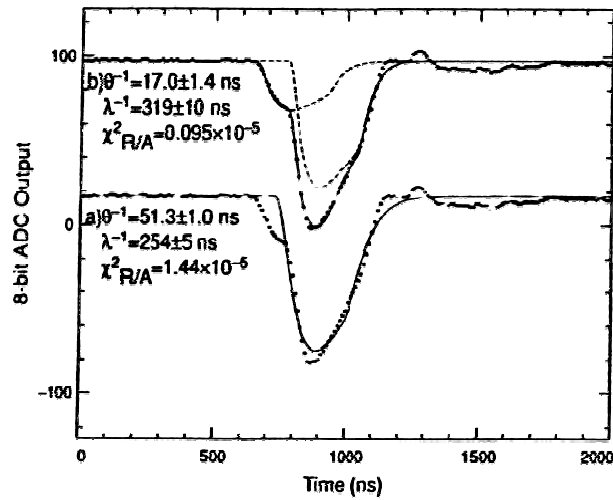


Figure 4.19. Demonstration of the deconvolution of pile-up pulses from the output of a scintillation counter. In the bottom plot, an analytic fit to these data using the assumption that a single pulse shape should suffice results in a poor fit. In the top plot, an iterative procedure was used to fit two pulse components that allow the separation of both from the pile-up recorded data (From Komar and Mark)

Combinations of the expected shape from two events with varying amplitude and time spacing can be fitted to the observed pulse shape, following an iterative process. Once the optimal fit is obtained, then the individual amplitudes of each event are obtained from the parameters of the fit and the mutual interference of the two events is eliminated. Not only is the corresponding distortion removed from the recorded pulse height spectrum, but now the full information carried by the two separated pulses can be obtained.

Although the utility of this type of pile-up deconvolution has been demonstrated in off-line systems the required processing times are still sufficiently long that is not yet standard routine in processing systems in which pulse analysis is performed.

The Experimental Problem

In Figure 4.20 an example of experimental pulse pile-up is presented. The data frame here presented constitutes a record of a valid pulse sensed by the pulse-detection

routine. Note that the arrival of one, maybe two photons occur during the count-down time¹¹.

When looking at the pulse-detection records as the one shown in Figure 4.20, it can be noticed that the phenomenon of pulse pile-up is present in a significant number of them. This represents a major problem when trying to obtain a good template of the pulse waveform, crucial to perform the rest of the digital processing algorithm and posterior analysis in a satisfying way. For this reason it is imperative to determine a method to reject pile-ups occurrence.

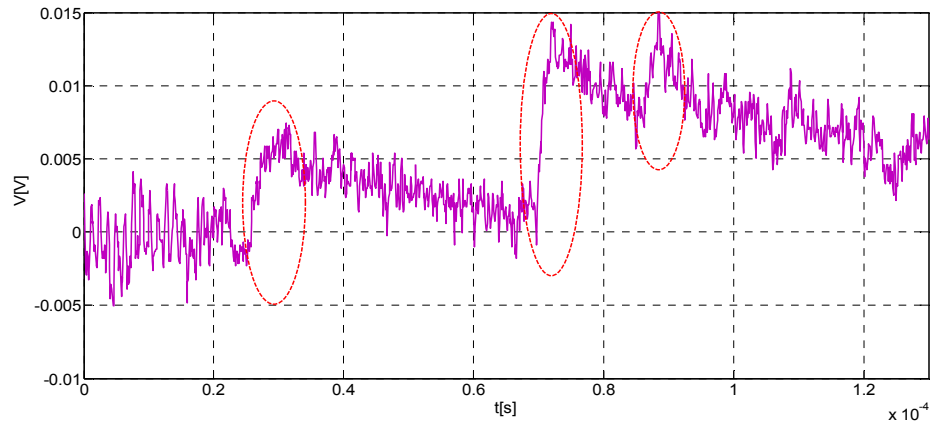


Figure 4.20. Record of a detected pulse with the presence of *tail Pile-up* phenomena

Other examples of selected data frames are presented in Figure 4.21, where the pile-up occurrence is evidenced.

¹¹ Interval of time necessary to count-down the length of a pulse data record (number of samples needed by the Pulse Height Analysis algorithm to calculate the pulse-height to the highest possible resolution)

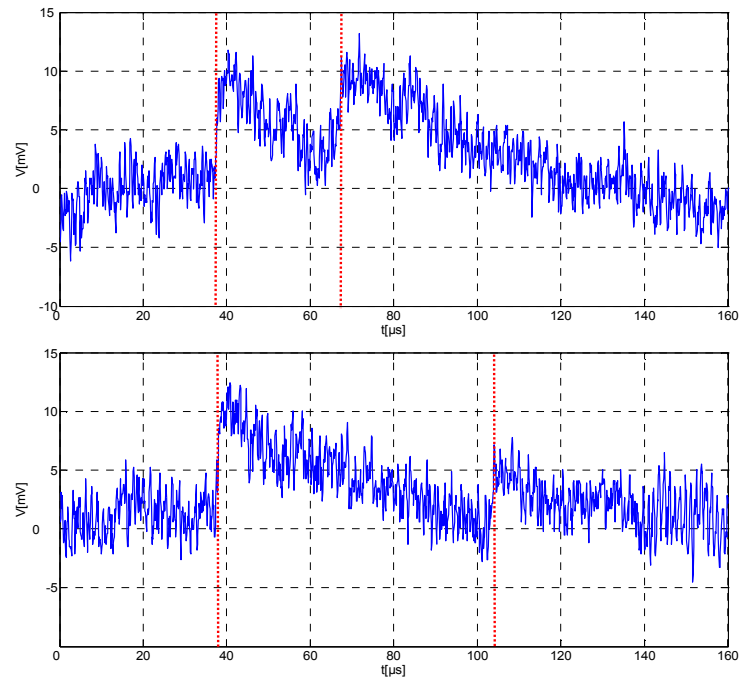


Figure 4.21. Pile-ups evidenced in the Pulse Data Records stored during the evaluation of the Pulse Detecting Algorithm

Pile-Up Rejection Algorithm

The routine developed to reject pile-up events, similar to the illustrated in Figure 4.21, is based on a recursive comparison between the energies of the pulses detected and the signal template which is recalculated every time a pile-up event is recognized and discarded. In Figure 4.22 the pile-up rejection algorithm is illustrated through a simplified flow-chart. For the full Matlab® routine see the Appendix.

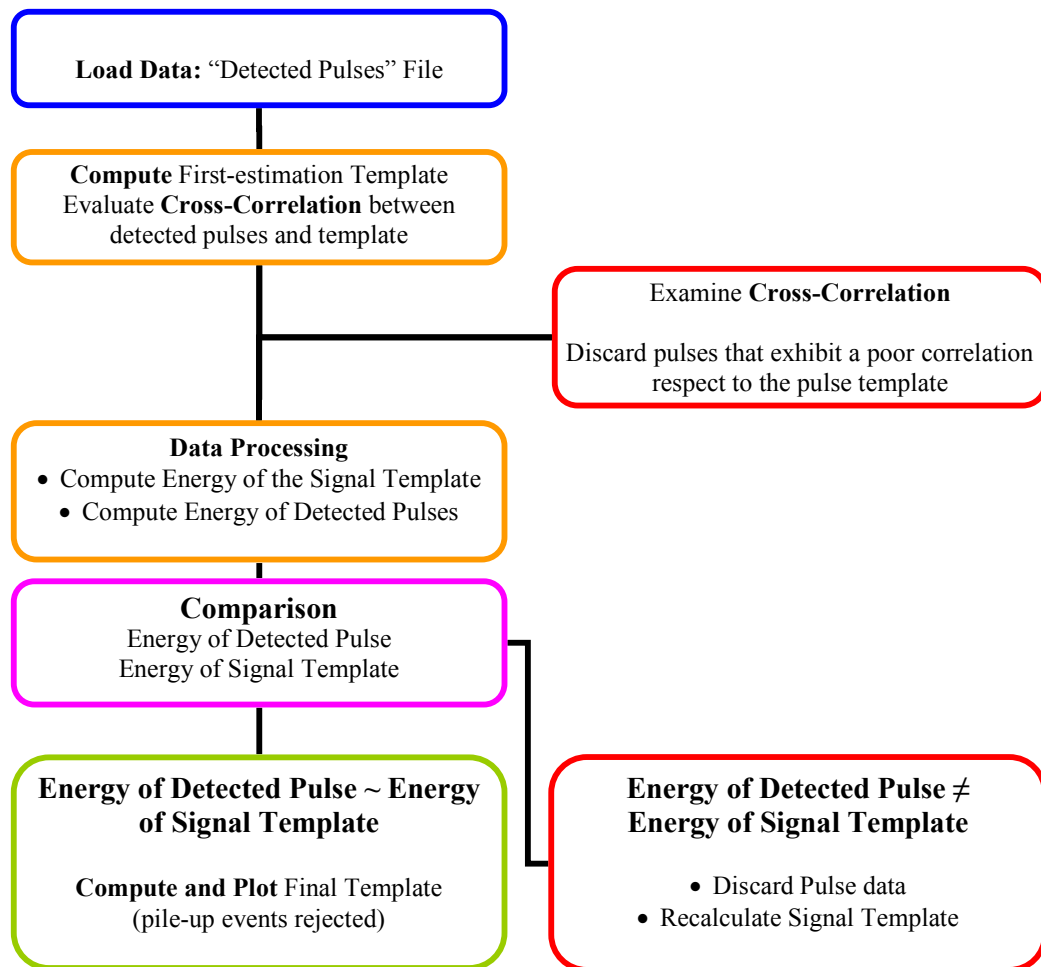


Figure 4.22. Pile-up Rejection Algorithm

In the next page, Figure 4.23 and Figure 4.24 show the pulses detected with and without the Pile-Up Rejection treatment. It can be seen how the pulses with pile-up are discarded. This directly influence the pulse template: in this case it lightly increases its peak-amplitude, although seems to be a little noisier than the original; this can be justified by the fact that less pulses are mediated in order to recalculate the signal template, because many of them are discarded when the presence of pile-ups is revealed.

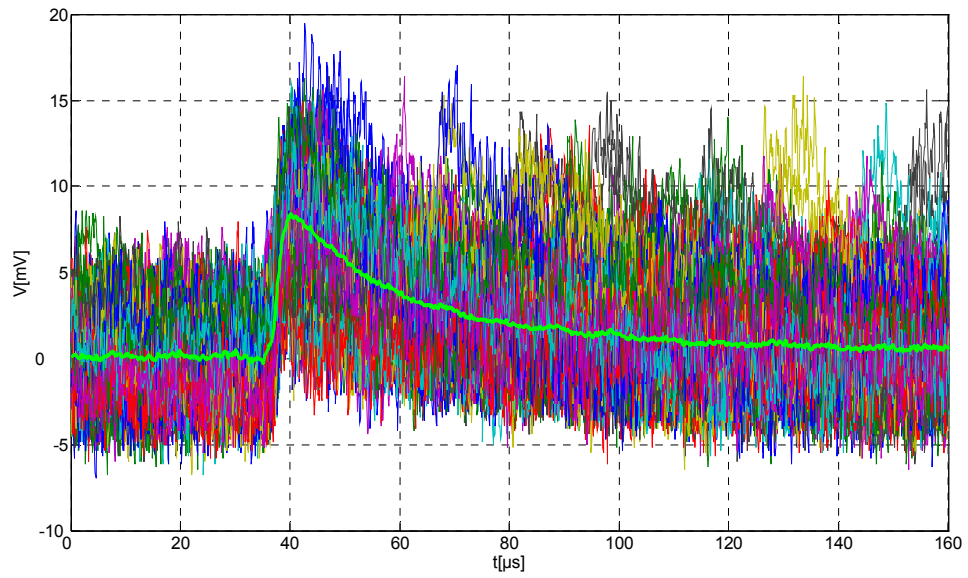


Figure 4.23. Detected Pulses with no Pile-up Rejection

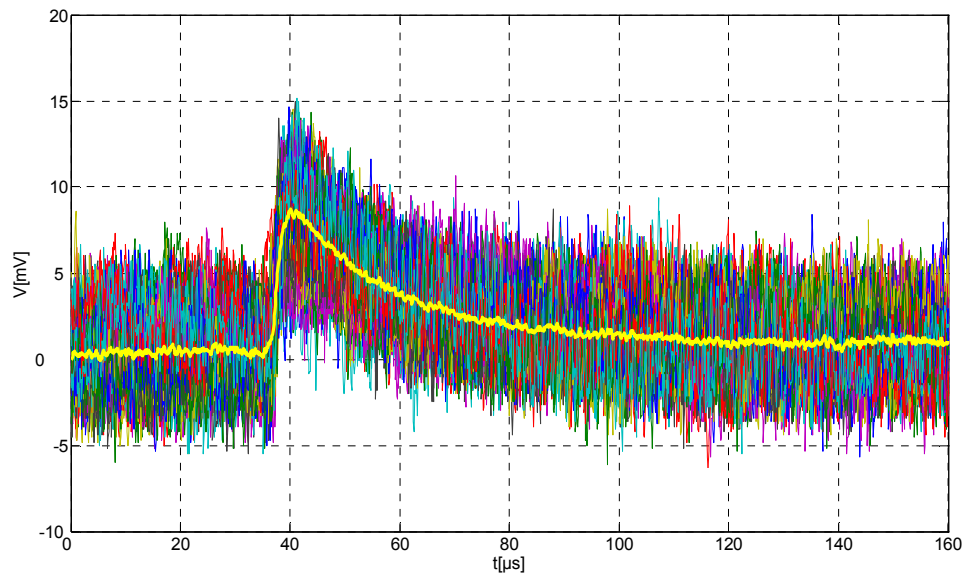


Figure 4.24. Detected Pulses with Pile-up Rejection

4.2.3 Digital Baseline Restoration

In order to make a good estimation of the amplitude of the pulses detected by the TES photodetector, it is necessary to perform a restoration of the baseline, which is just the average level of the signal just before the occurrence of a detected pulse.

In digital pulse-processing systems, the baseline restoration process can be accomplished by digitally sampling the baseline between pulses and then subtracting an appropriated value from the measured pulse amplitude.

Independent baseline estimates can be obtained for each pulse that is processed, or alternatively, a common estimated applied to a series of pulses between more widely spaced samples of the baseline. It would be ideal having multiple measurements of the baseline taken for a given sample to enhance its accuracy, but this would require a relatively long time interval between pulses. Therefore a trade-off must exist between the accuracy of the baseline measurement and the frequency with which they can be made.

In this work common baseline estimation is made for the pulses detected, from every pulse record a significant number of samples before the generated trigger were stored for this calculation. This way is possible to perform a more accurate calculation on the pulses height.

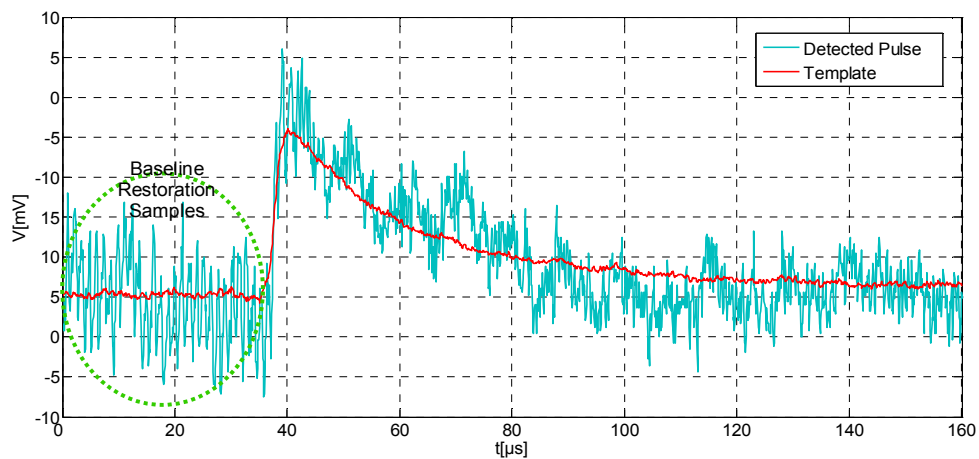


Figure 4.25. An example of Baseline Restoration Samples stored within a Pulse Detected Record

4.2.4 Pulse Height Estimation

Once the pulses have been detected and processed in order to eliminate pile-up events and restore the baseline, the last step of the PHA algorithm consists on estimating the amplitude of the pulses detected. This calculation is performed by a very simple routine developed in Matlab® which estimates the pulse height by subtracting the corresponding baseline value (previously estimated), to the maximum amplitude reached by the signal.

4.3 Signal Filtering

Once the data has been processed in order to detect the pulses and reject the phenomenon of pile-up, then the elaboration of the signal continues in order to obtain a good templates of the pulse shapes.

Until now, the algorithm presented has done nothing but processing the data with the intention of emulate the conditions that would exist in the case of a triggered light emitting source. From now on, the processing routine will be the same for the two cases of study: triggered and non triggered sources.

With the aim of achieving a good quality signal different kinds of digital filters were applied to the signal: low pass filter, notch filters, and matched filters.

4.3.1 Digital-Signal-Processing Filters

As explained in Chapter 2, a digital filter computes a quantized time-domain representation of the convolution of the sampled input time function and a representation of the weighting function of the filter. They are realized by an extended sequence of multiplications and additions carried out at a uniformly spaced sample interval. Simply said, the digitized input signal is mathematically influenced by the DSP program. These signals are passed through structures that shift the clocked data into summers (adders), delay blocks and multipliers. These structures

change the mathematical values in a predetermined way; the resulting data represents the filtered or transformed signal.

In this work, filtering of the signals is performed in practically every step of the DSP algorithm. This section is dedicated to explain the different filters used along the digital processing algorithm.

4.3.2 Pre-Filtering

When filtering the signals through a linear filter, drift and high-frequency disturbances in the data can be removed, that should not affect the model estimation. Pre-filtering is a good way of removing high-frequency noise in the data, and is also a good alternative to detrending¹² (by cutting out low frequencies from the pass band). The pre-filtering is however valid only for estimating the dynamics from input to output. The disturbance model is determined from the original data.

With the objective of performing an adequate pre-filtering is necessary to analysis the spectral content of the signals. Once this has been done, the parameters of the filter can be setup at adequate levels.

The data frames stored during the acquisition phase have a finite duration in time; therefore, the lower frequency of the signals that can be analyzed corresponds to $f_{\min} = 1 / t_{\text{frame}}$. This could be consider as a high-pass filtering of the signals, since it rejects all frequencies bellow $f_{\min}$; for this reason no high-pass filtering was applied to the signal data.

On the other hand, given that the sampling rate chose to perform this DSP routine has a value of 10MHz, the effective spectral content of the signals that will be

¹² *Trend* in a time series is a slow, gradual change in some property of the series over the whole interval under investigation. Trend is sometimes loosely defined as a long term change in the mean, but can also refer to change in other statistical properties. *Detrending* is the statistical or mathematical operation of removing trend from the series

analyzed corresponds to a bandwidth of 5MHz according to the Sampling Theorem. As a result, the low-pass filters applied to the signals have cut-off frequencies that are minor that 5MHz, otherwise the filtering will be worthless.

In order to perform this pre-filter routine the Matlab® Digital Filter Design Tool was employed. Depending on the signal, different cut-off frequencies were established to perform a low-pass filtering of all the data.

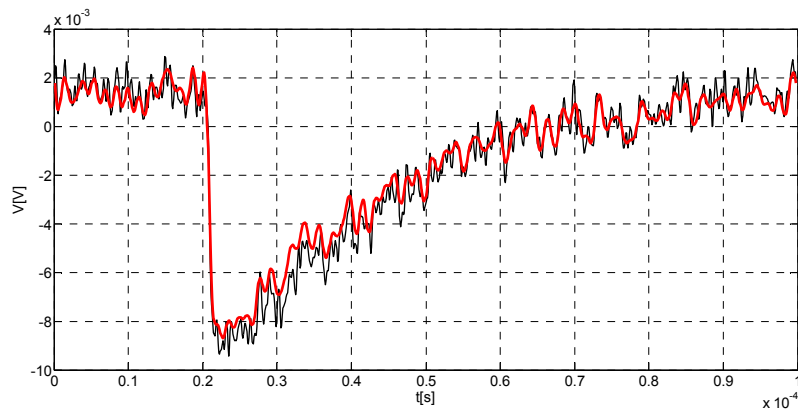


Figure 4.26. Example of a Pre-filtered Signal Data Frame with a Low-Pass Filter

4.3.3 Notched Filters

A Notch filter is a filter that passes all frequencies except those in a stop band centred on a frequency. In this work, notch filtering is used in order to eliminate some particular noise contributions present among the signals been processed.

Figure 4.27 shows the signal generated by the monochromator after the application of the notch filtering. The pre-filtrated signal presented an oscillating behaviour, both before the front edge of the pulse and during the decay time. When applying notch filters to this signal, properly centred in frequency, it is possible to eliminate this oscillation.

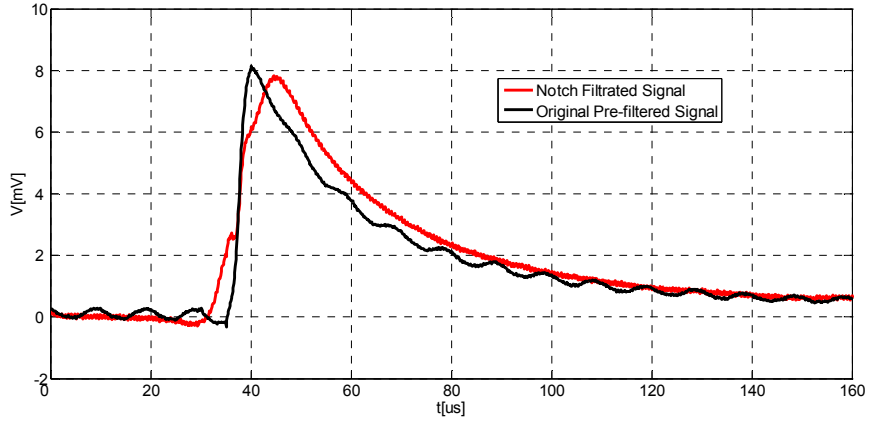


Figure 4.27. Notch Filtering applied to the signal from the monochromator

As usual, a trade-off has to be made when processing the signals with digital filters: notice that the oscillating behaviour is eliminated at the same time that the pulse height is affected together with the velocity of the front edge of the pulse.

In order to perform this filtering of the signals is necessary to analyze their power spectral content. In Figure 4.28 the power spectrums of the signal generated by the monochromatic source are presented, both with and without applying the notch filtering. In this case, the frequencies filtered are $f_1 = 0.1025MHz$ and $f_2 = 1.567383MHz$.

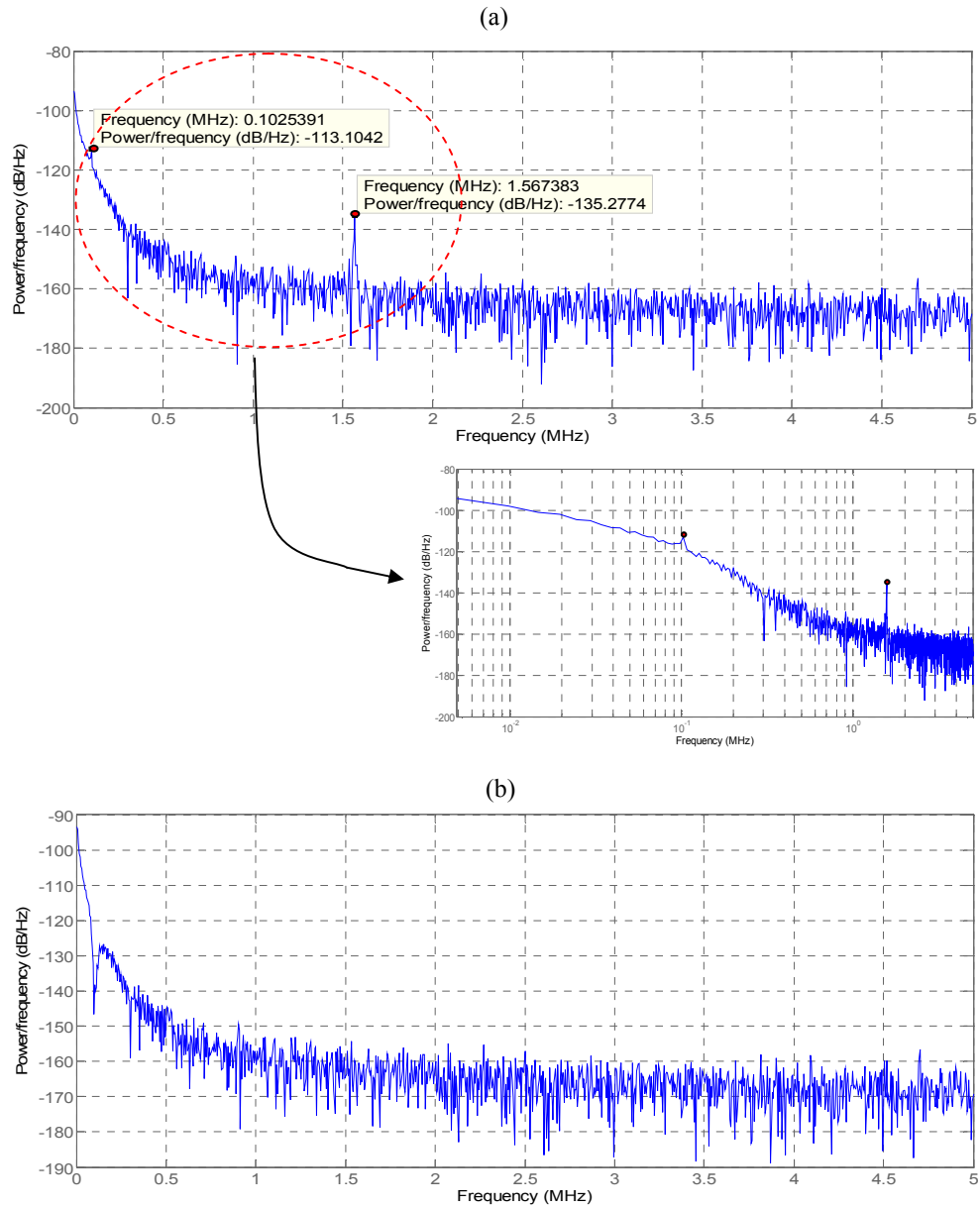


Figure 4.28. Notch filtering applied to the signals generated by the Monocromator. (a) Power Spectral Density of the original signals. (b) Power Spectral Density of the filtrated signals

4.3.4 Matched Filter

A *matched filter* is obtained by correlating a known signal, or template, with an unknown signal to detect the presence of the template in the unknown signal. This is equivalent to convolving the unknown signal with a time-reversed version of the template (cross-correlation). The matched filter is the optimal linear filter for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise.

Implementation of the matched filter

In this work, the matched filter is used for two main purposes:

- As part of the pile-up rejection algorithm, to calculate the correlation existing between the pulses detected by the PHA routine and the estimated pulse shape
- To filtrate the pulses detected by the PHA routine in order to eliminate the high level of noise present on them.

After the pulse detection has been performed, a first estimated of the pulses shape is obtained: the events detected by the pulse-detection are mediated in order to obtain the pulse template. This first estimation is assumed to be the first approximation of the matched filter.

During the application of the pile-up rejection routine an inspection of the pulses detected by the PHA routine is made. Since the signals obtained when using the monochromator exhibit elevate noise levels, sometimes the derivate reveals the presence of *fake pulses* -caused by the high frequency noise-, and the pulse detection routine stores them as valid events. The correlation between the detected pulses and the matched filter is calculated then, in order to determine if the pulses are effectively generated by the absorption of a photon in the TES detector, or instead they are caused by a false trigger produced by the elevate high-frequency noise content of the signal.

The matched filter is also used to eliminate the noise from the signals acquired. Figure 4.29 and Figure 4.30 show the pulses templates obtained after filtering the signals acquired and mediating them, both when employing the monochromator and the laser diode source.

The initial approximation of the matched filter is obtained by mediating all pulses detected without any processing. Then, a recursive filtering of the signals is done with the new pulse template which is recalculated every time the whole set of data is filtered with the previous matched filter estimated.

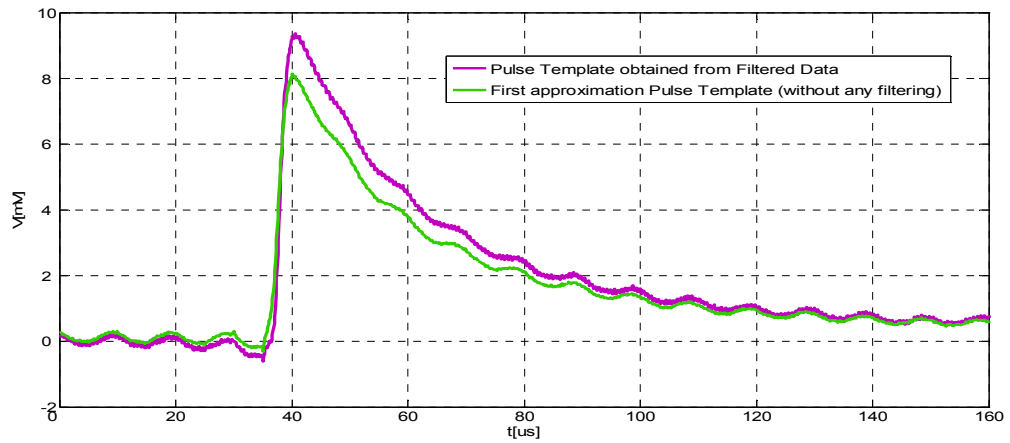


Figure 4.29. Pulse Templates obtained with and without matched filtering: Monochromatic Source

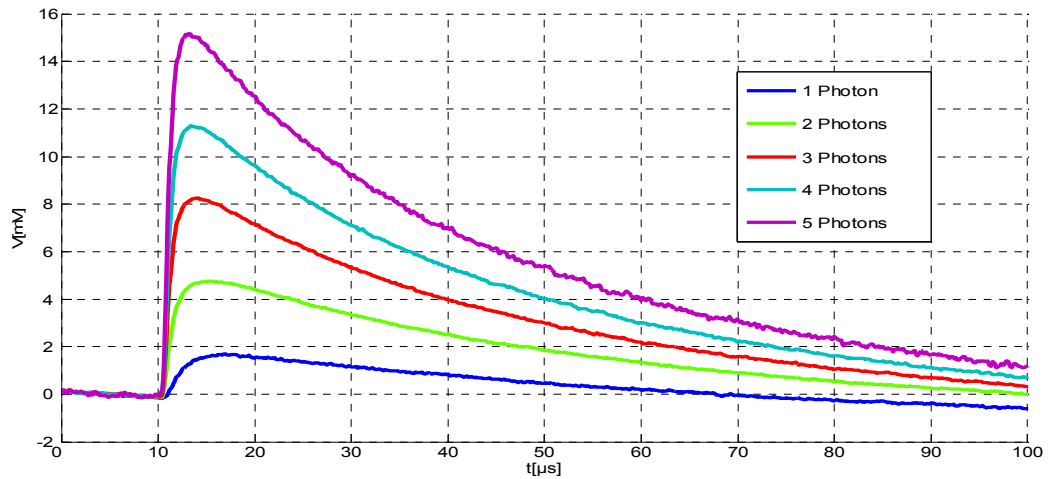


Figure 4.30. Pulses Templates obtained with matched filtering: Laser diode Source.

4.4 Signal Histograms

Figure 4.31 resumes the DSP algorithm proposed in this thesis through a flow-chart. This Section is dedicated to describe the final step of the algorithm: Computing Signal Histograms.

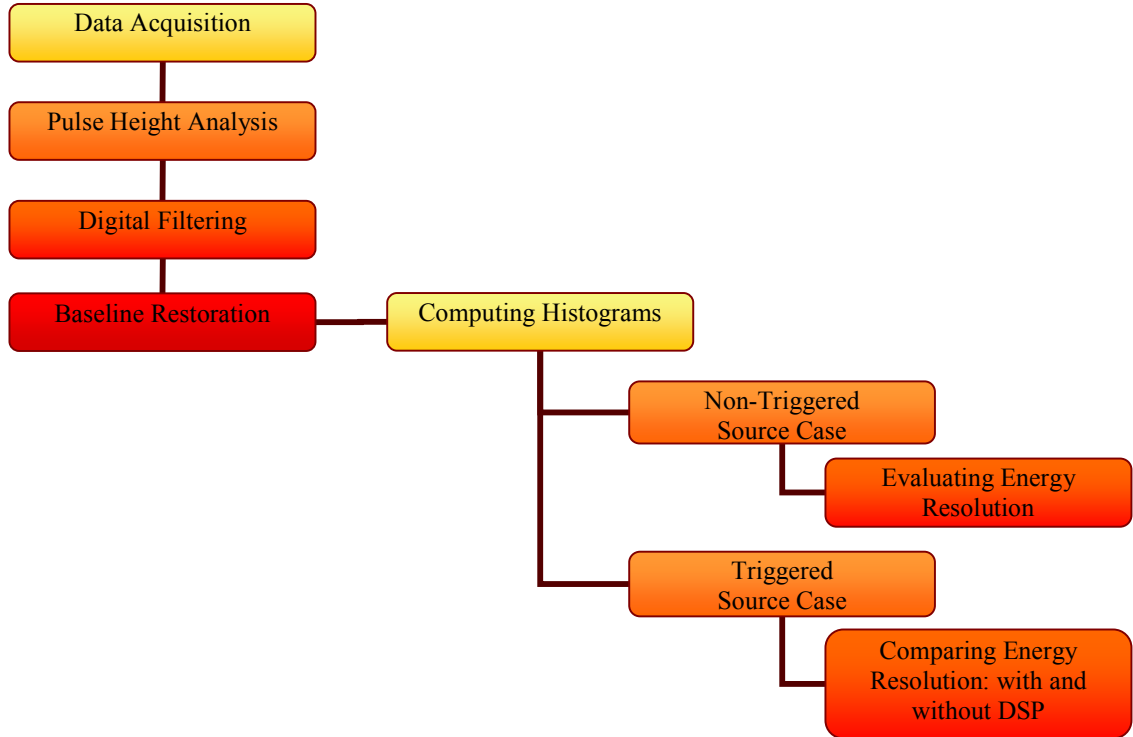


Figure 4.31. Digital Signal Processing Algorithm

Once the pulse amplitudes have been calculated restoring the baseline of the signal, it is possible to compute the different signal histograms. In order to obtain this graphics it is necessary plotting the amplitude of the pulses from the detected signal, and their corresponding number of counts.

One important property of the single-photon detectors systems is the energy resolution. This concept is most easily illustrated by assuming that the detector is exposed to radiation quanta of a single fixed energy. Many radiation quanta then deposit the same energy in the detector and ideally should produce exactly the same charge Q . Therefore, a number of pulses of precisely the same amplitude should be

presented to the PHA routine and stored and classified as part of a pulse group with common characteristics.

In actual systems, however, some fluctuations are observed in the amplitude of these pulses, and they are actually spread out over a number of channels in the spectrum, as illustrated in Figure 4.32

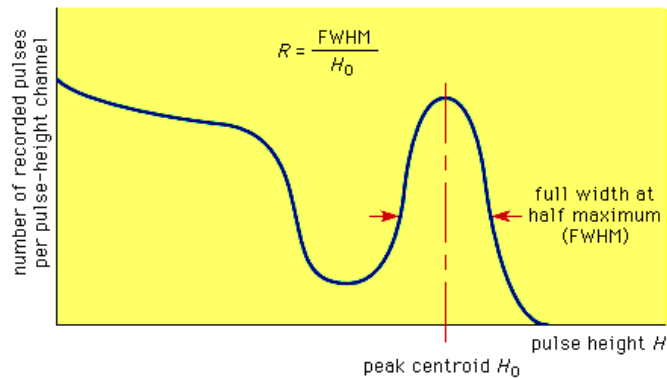


Figure 4.32. Energy Resolution and Full Width at Half Maximum

A formal definition of energy resolution is shown in the figure, expressed as the ratio of the full-width-at-half-maximum (FWHM) of the peak divided by the centroid position of the peak.

This ratio is normally expressed as a percentage, and small values correspond to narrow peaks and good energy resolution. If the incident radiation consists of multiple discreet energies, good energy resolution will help in separating the resulting peaks in the recorded pulse-height spectrum.

Some potential causes of fluctuations that broaden the peaks include drifts in the detector-operating parameters over the course of the measurement, random fluctuations introduced by the noise in the pulse-processing electronics, and statistical

fluctuations due to the fact that the charge Q consists of a finite number of charge carriers¹³.

This latter statistical limit is in some ways the most fundamental determinant in energy resolution since, as opposed to the other sources of fluctuation, it cannot be reduced by more careful experimental procedures.

Poisson statistics predicts that the fractional standard deviation that characterizes these fluctuations about the average number of charge carriers N should scale as $1/\sqrt{N}$. Therefore, detectors that produce the largest number of carriers per pulse show the best energy resolution.

The histograms allow evaluating the energy resolution of the Superconducting TES single-photon detector. As shown in Figure 4.32 the energy resolution (R) is given by the formula:

$$R = FWHM / H_o$$

where FWHM is the Full Width at Half Maximum and H_o is the peak centroid of each peak.

When considering the normal distribution function, with a standard deviation σ , the FWHM is defined by:

$$FWHM = 2\sigma\sqrt{2\ln 2} \approx 2.355\sigma$$

When dealing with "particles" such as photons or electrons, a commonly used unit of energy is the electron-volt (eV) rather than the joule (J). An electron volt is the energy required to raise an electron through 1 volt, thus $1 \text{ eV} = 1.602 \cdot 10^{-19} \text{ J}$.

A photon is characterised by either a wavelength, denoted by λ and an equivalently energy, denoted by E . There is an inverse relationship between the

¹³ In physics, a charge carrier denotes a free (mobile, unbound) particle carrying an electric charge. Examples are electrons and ions. In semiconductor physics, the travelling vacancies in the valence-band electron population (holes) are treated as charge carriers.

energy of a photon (E) and the wavelength of the light (λ) given by the equation:
 $E = hc / \lambda$, where h is Planck's constant ($h=6.626 \times 10^{-34}$ J·s) and c is the speed of light.

For the two cases considered in this thesis, the energies of a photon are:

- Monochromator ($\lambda=200\text{nm}$): 6.2eV
- Laser Diode Source ($\lambda=690\text{nm}$): 1.7971eV

4.4.1 Non-triggered Source Case

When working with the Monochromatic Source, two data sets were considered in order to compute the corresponding histograms: the detected pulses with and without applying the pile-up rejection routine.

In Figure 4.34, the fitting corresponding to the pulse height histograms of the considered case are presented.

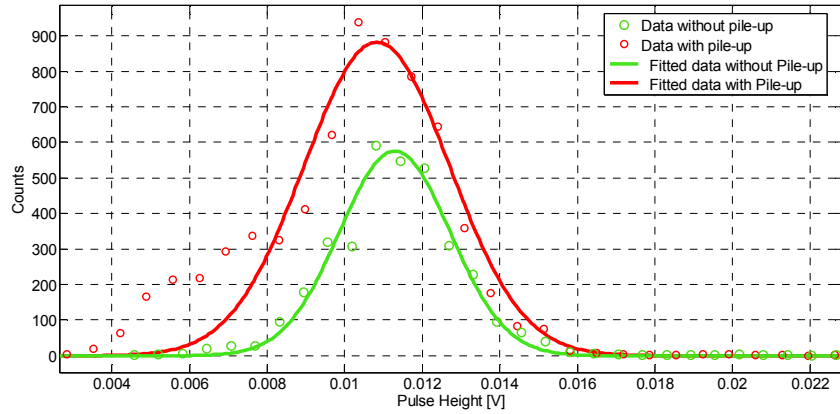


Figure 4.33. Histograms of the pulses detected generated by the Monochromatic source.

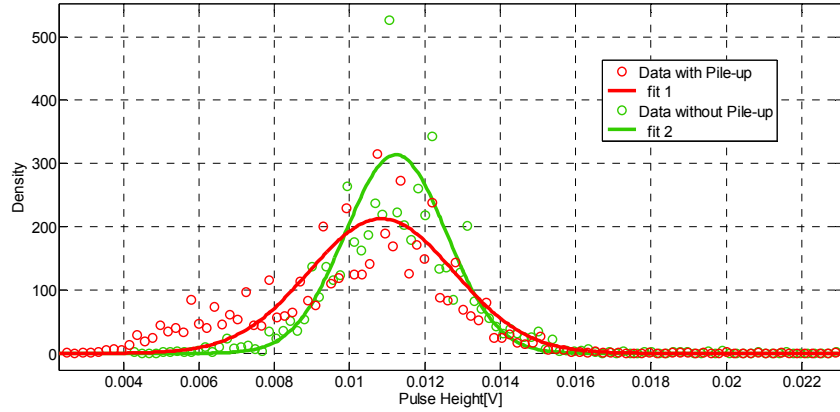


Figure 4.34. Probability density functions of the Signals from the Monochromatic Source

In Figure 4.34, the sample probability functions are shown, for both the cases in which the pile-up rejection routine is applied or not.

It can be clearly seen how the rejection of pile-ups is traduced in a graphical improvement of the Gaussian that represents the probability distribution of the pulse heights, the peak becomes narrower and the mean lightly increases its value.

The values of the pulse heights (Gaussian mean) and the FWHMs (proportional to the standard deviation) of the fitted curves from Figure 4.34 are reported on Table 4-1 along with the values corresponding to the respective energy resolution

	Pulse Height[mV]	FWHM[mV]	Energy Resolution
Pile-up Rejection	11.23	1.341	11.94%
NO Pile-up Rejection	10.84	1.922	17.73%

Table 4-1. Energy Resolution of the TES device whit and without Pile-up rejection applied to the signal from the monochromatic source.

In this case is hard to establish a comparison, due to the fact that both data sets are been partially or totally processed with the different DSP routines proposed until now. However, it is clear, that the energy resolution is improved when applying the Pile-up Rejection routine to the data been processed. Graphically the peak of the fitted Gaussian becomes narrower, which is traduced in a reduction of the standard deviation value.

4.4.2 Triggered Source Case

In Figure 4.35 and Figure 4.36 the histograms obtained when working with the laser triggered source are presented. The two data sets considerer correspond to the processed data with the DSP algorithm and the data obtained directly from the detector.

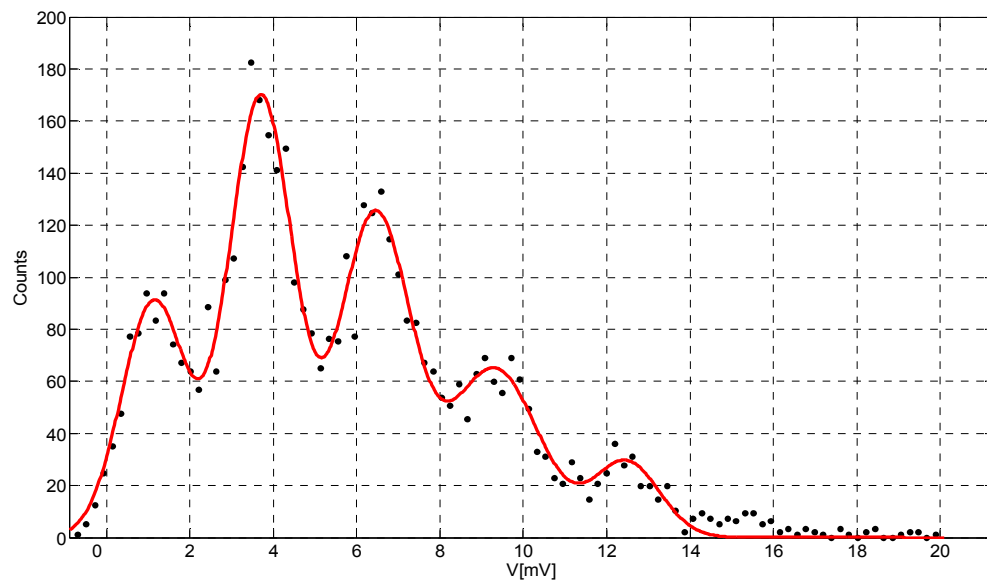


Figure 4.35. Signal Histogram without applying DSP routine: Triggered Source

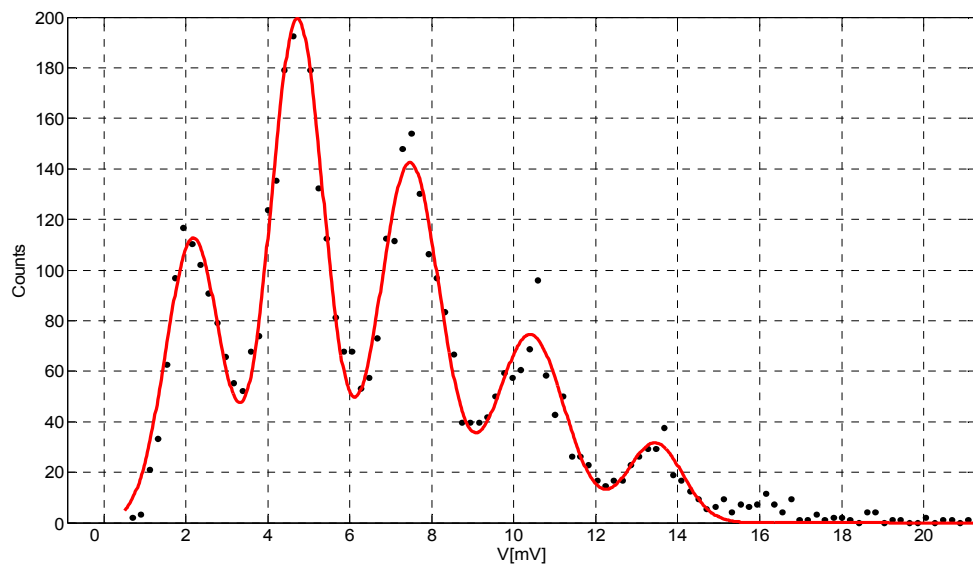
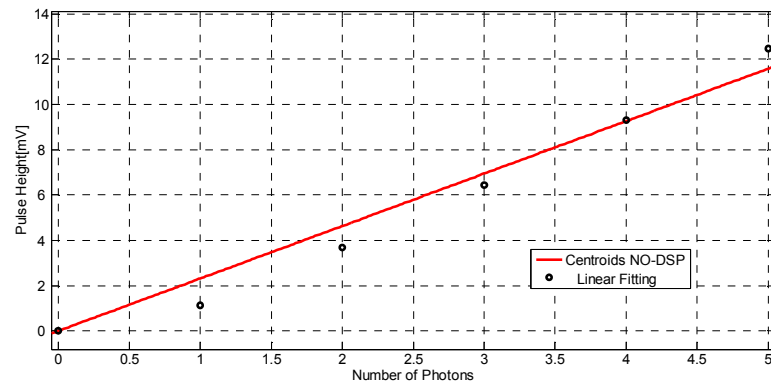
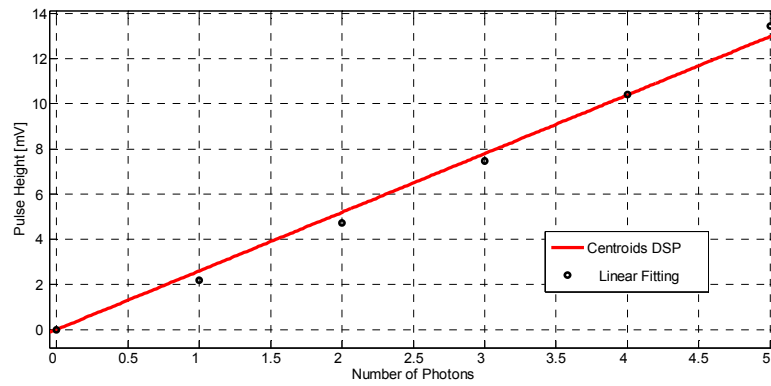


Figure 4.36. Signal Histogram applying DSP routine: Triggered Source

In Figure 4.37 the values of the pulse height both when using the triggered and the non-triggered source are shown, along with the corresponding linear fitting.



(a) Unprocessed Data



(b) Processed Data

Figure 4.37. Linear fitting of the Pulses Heights for the processed and unprocessed data histograms.

In general, notice that the pulse heights values show a quite linear relationship between them. However, is clear that, with data processing, the linear relationship between the pulses heights improves respect to the case when no DSP is applied.

Similarly to the monochromatic source case, a table is presented where the parameters of interest are reported. In this case it is necessary to consider all five Gaussians. The values corresponding to the FWHM and the energy resolution of the two Histograms are presented on Table 4-2

		Pulse Height	FWHM	Energy Resolution(%)
NO DSP	1 Photon	0.001132	0.0018265	161.3
	2 Photons	0.003699	0.0018211	49.232
	3 Photons	0.006437	0.0020173	31.336
	4 Photons	0.009305	0.0024917	26.777
	5 Photons	0.012467	0.0018731	15.024
DSP	1 Photon	0.002175	0.0015736	72.325
	2 Photons	0.004717	0.0015583	33.037
	3 Photons	0.007463	0.0017677	23.684
	4 Photons	0.010400	0.0019772	19.011
	5 Photons	0.013445	0.0016015	11.911

Table 4-2. Energy Resolution of the TES device when using the triggered source

In this case, the improvement of the energy resolution of the signal is evident when comparing the results obtained with and without the DSP algorithm. In the histograms can be appreciated how all of the five peaks become much narrower when processing the data.

CONCLUSIONS

Many scientific applications like astronomy, material analysis, metrology, together to quantum information and quantum computation need microcalorimeters with high quantum efficiency (QE), high energy resolution and photon number capability for the measurement of visible-near infrared single photons.

In this sense TESs are innovative cryogenic devices that exploit their sharp transition from the superconducting to normal state to detect single photon, that have demonstrate to have outstanding capabilities; good energy resolution and photon number discrimination at visible and near-infrared wavelengths make them used as single photon detectors.

Among the different TES's that have been fabricated by the "Cryogenic Innovative Devices" group at the INRiM, a Pd/Ti TES has been characterized as a single photon detector.

The signal obtained from this detector, object of study of this thesis, has been digitally processed in order to evaluate and improve the energy resolution of the TES single photon detector.

The DSP algorithm proposed in this work has demonstrated to improve the energy resolution of the signal obtained from the TES single-photon detector.

The first stage of the DSP algorithm, has verified a good performance detecting the pulses in the signal data received by the TES detector that has been generated by a monochromatic source. Then, the different type of digital filtering applied to the signal has clearly improved its quality, rejecting high-frequency noise and systematic interferences. At last, a good estimation of the pulse shape was obtained. In the case of the monochromatic source the only event observed was the arrival of a single photon to the detector, while for the triggered source (Laser) different templates that correspond to the events of the arrival of one single photon to the simultaneously arrival of five photons were obtained.

All of these manipulations of the signals have allowed obtaining an improved energy resolution that can be appreciated in the computed histograms, where a reduction on the value of the FWHM of the peaks, corresponding to the events of the arrival of one or more photons, can be observed.

Computer-based software routines developed in this thesis allows the user to experiment with DSP; non-real-time signal processing can be performed varying the parameters of each stage which traduces in a wide range of possibilities and gives the algorithm a universal character: it could be adapted for any type of single-photon detector.

At last, some suggestions for future work can be made in order to continue improving the performance of the proposed algorithm.

Regarding pile-up events, nowadays, the availability of digital pulse-processing systems opens the possibility of doing more than simply tolerating the losses associated with pile-up. If a detailed digital sampling is made of each output pulse shape it should in principle be possible to distinguish those that originate from a single event from those involving pile-up in which more than one event have

contributed. The possibility then arises of deconvolving the piled-up pulses into their separate components through a detailed analysis of their shape.

Finally, most DSP algorithms are not very complicated. In fact, some of the most important DSP functions are implemented by a simple repeated multiplication and summation operation, and could be written out in a basic program that nearly anyone could understand. However, the speed with which the algorithms must operate often requires specialized microprocessors ("DSP chips") in order to run in real-time.

In this sense, the computer-based software developed in this thesis can be used to generate the real-time software that will run on a dedicated DSP microprocessor.

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APPENDIX

MATLAB® ROUTINES

DSP Triggered Source

DSP.m

```
%% Digital Signal Processing Algorithm: Triggered Source
clc
clear all
tic
%% File reading
nfile=input('Number of files to be analyzed: ');
as=zeros(998,nfile-1);
for n=0:1:nfile-1
    snumber=int2str(n);
    if (n<10)
        s='0000';
        snumber=[s snumber];
    else if (n<100)
        s='000';
        snumber=[s snumber];
    else if (n<1000)
        s='00';
        snumber=[s snumber];
    else
        snumber=[int2str(0) snumber];
    end
    end
end

    signalfile=['C:\MATLAB7\work\30uA\misure_luglio\C2690nm_B-
0_8mA_55_4mK_30uA_4_1ND' snumber '.txt'];
    %signalfile=['C2690nmIold20uAND5_8B0_47_7mK' snumber
'.txt'];
```

```

        %signalfile=['C:\Documents and
Settings\Administrator\Documenti\DATI\Diluizione\Anno2007\Novembre\L
eCroy_19_23Novembre07\1fotone_amp\c2\C2_1fot_38uA_nd5_9' snumber
'.txt'];
        [ts(:,1),as(:,n+1)]=readData_it(signalfile,5);
end

%% Data Processing
as_bl=mean(as(1:200,:));
                                %baseline before the pulse 200 campioni
mat_bl= repmat(as_bl,size(as,1),1);
as_new=as-mat_bl;
                                %pulse with zero baseline

figure
hold on
for i=1:nfile
    plot(ts,as_new(:,i));
    hold all
end
% save C2690nm_B-0_8mA_55_4mK_30uA_4_1ND.dat as_new -ascii
%save C2_2fot_38uA_nd5_9.dat as_new -ascii

%as_min=-min(as(201:350,:));
                                %trova il minimo dell'impulso

%%
prova=passabasso(as);
%%
media_filt=mean(prova(1:200,:));
mat_bl= repmat(media_filt,size(prova,1),1);
prova_new=prova-mat_bl;
                                %pulse with zero baseline

figure
hold on
for i=1:nfile
    plot(ts,prova_new(:,i));
    hold all
end
save C2690nm_B-0_8mA_55_4mK_30uA_4_1ND.dat as_new -ascii
save C2_2fot_38uA_nd5_9.dat as_new -ascii

%with DSP
as_min=-min(prova_new(201:300,:));
figure
hist(as_min,70)
[x,y]=histogram(as_min);
gauss_fit(x,y)
grid on

%without processing
as_min_nodsp=-min(as(201:300,:));
figure
hist(as_min_nodsp,70)
[xno,yno]=histogram(as_min_nodsp);
gauss_fit(xno,yno)

```

```
grid on
```

```
toc
```

Functions used by DSP.m

histogram.m

```
function [C_,N_]=histogram(as_min)

%   C_ corrisponde ai valori "x"
%   N_ corrisponde ai valori "y"

%HISTOGRAM    Create plot of datasets and fits
%   HISTOGRAM(AS_MIN)
%   Creates a plot, similar to the plot in the main distribution
fitting
%   window, using the data that you provide as input.
%
%   Number of datasets:  1
%   Number of fits:     0

% Data from dataset "as_min data":
%   Y = as_min

% Remove missing values
t_ = ~isnan(as_min);
as_min = as_min(t_);

% Set up figure to receive datasets and fits
figure
f_ = clf;
figure(f_);
legh_ = []; legt_ = {}; % handles and text for legend
ax_ = newplot;
set(ax_,'Box','on');
hold on;

% --- Plot data originally in dataset "as_min data"
as_min = as_min(:);
[F_,X_] = ecdf(as_min,'Function','cdf'...
               ); % compute empirical cdf
Bin_.rule = 3;
Bin_.nbins = 100;
[C_,E_] = dfswitchyard('dfhistbins',as_min,[],[],Bin_,F_,X_);
[N_,C_] = ecdfhist(F_,X_,'edges',E_); % empirical pdf from cdf
h_ = bar(C_,N_,'hist');
set(h_,'FaceColor','none','EdgeColor',[0.333333 0 0.666667],...
      'LineStyle','-','LineWidth',1);
xlabel('Data');
ylabel('Density')
legh_(end+1) = h_;
legt_{end+1} = 'as_min data';

% Nudge axis limits beyond data limits
```

```

xlim_ = get(ax_,'XLim');
if all(isfinite(xlim_))
    xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
    set(ax_,'XLim',xlim_)
end

x_ = linspace(xlim_(1),xlim_(2),100);

hold off;
legend(ax_,legh_, legt_, 'Location','NorthEast');

save hist_x.dat C_ -ascii
save hist_y.dat N_ -ascii

```

gauss_fit.m

```

function gauss_fit(hist_x,hist_y)
%load hist_x.dat
%load hist_y.dat

%GAUSS_FIT      Create plot of datasets and fits
%  GAUSS_FIT(HIST_X,HIST_Y)
%  Creates a plot, similar to the plot in the main curve fitting
%  window, using the data that you provide as input.

%  Number of datasets:  2
%  Number of fits:     1

% Data from dataset "hist_y vs. hist_x":
%  X = hist_x:
%  Y = hist_y:
%  Unweighted

% Data from dataset "hist_y vs. hist_x with hist_y":
%  X = hist_x:
%  Y = hist_y:
%  Weights = hist_y:

% Set up figure to receive datasets and fits
f_ = clf;
figure(f_);
legh_ = []; legt_ = {}; % handles and text for legend
xlim_ = [Inf -Inf]; % limits of x axis
ax_ = subplot(1,1,1);
set(ax_,'Box','on');
axes(ax_); hold on;

% --- Plot data originally in dataset "hist_y vs. hist_x"
hist_x = hist_x(:);
hist_y = hist_y(:);
% This dataset does not appear on the plot

```

```

% Use this statement if the data is not weighed
% Add it to the plot by removing the if/end statements that follow
% and by selecting the desired color and marker
if 0
    h_ =
line(hist_x,hist_y,'Color','r','Marker','.', 'LineStyle','none');
    xlim_(1) = min(xlim_(1),min(hist_x));
    xlim_(2) = max(xlim_(2),max(hist_x));
    legh_(end+1) = h_;
    legt_{end+1} = 'hist_y vs. hist_x';
end % end of "if 0"

% --- Plot data originally in dataset "hist_y vs. hist_x with
hist_y"
h_ = line(hist_x,hist_y,'Parent',ax_,'Color',[0 0 1],...
    'LineStyle','none', 'LineWidth',1,...
    'Marker','.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(hist_x));
xlim_(2) = max(xlim_(2),max(hist_x));
legh_(end+1) = h_;
legt_{end+1} = 'Histogram';

% Nudge axis limits beyond data limits
if all(isfinite(xlim_))
    xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
    set(ax_,'XLim',xlim_)
end

% --- Create fit "fit 2"
fo_ =
fitoptions('method','NonlinearLeastSquares','Weights',[2.0811871
3.1217807 20.811871 33.298994 62.435613 96.775201 116.54648
110.30292 101.97817 90.531639 79.08511 65.557394 55.151458 52.029678
67.638581 73.882142 123.83063 135.27716 178.98209 192.50981
212.28109 178.98209 132.15538 112.3841 81.166297 67.638581 67.638581
53.070271 57.232646 72.841549 112.3841 111.34351 147.76428 154.00785
130.07419 106.14054 96.775201 83.247484 66.597987 39.542555
39.542555 39.542555 41.623742 49.948491 59.313833 57.232646
60.354426 68.679175 95.734607 58.273239 42.664336 49.948491
26.014839 26.014839 22.893058 16.649497 14.56831 16.649497 16.649497
22.893058 26.014839 29.13662 29.13662 37.461368 18.730684 16.649497
12.487123 9.365342 5.2029678 6.2435613 9.365342 4.1623742 7.2841549
6.2435613 7.2841549 11.446529 7.2841549 4.1623742 9.365342 1.0405936
1.0405936 3.1217807 1.0405936 2.0811871 2.0811871 1.0405936 0
4.1623742 4.1623742 0 1.0405936 1.0405936 0 0 2.0811871 0 1.0405936
1.0405936 0 1.0405936 ],'Lower',[-Inf -Inf 0 -Inf -Inf 0 -Inf -
Inf 0 -Inf -Inf 0 -Inf -Inf 0 ]);

%-----
%-----
%-----
%-----

```

```

%-----
%-----
%Initial Coefficients (optimized for july data)
st_ = [116.546 0.002 0.0002 212.281 0.0045 0.0002 154 0.0075 0.0002
96 0.0105 0.0002 37.46 0.0135 0.0002 ];
%-----
%-----
%-----
%-----
%-----

set(fo_,'Startpoint',st_);
set(fo_,'Weight',hist_y);
ft_ = fittype('gauss5' );

% Fit this model using new data
cf_ = fit(hist_x,hist_y,ft_ ,fo_);
%coeff_mat = cf_.coeffValues;

% Or use coefficients from the original fit:
if 0
    cv_ = {112.4670363963, 0.002175670692683, 0.0009450174911593,
199.4019043782, 0.004717018789095, 0.000935877420562,
142.4147882158, 0.007463492692134, 0.001061552013281,
74.35691062117, 0.01040018621082, 0.001187440691246, 31.49130122111,
0.01344548276867, 0.0009617836923026};
    cf_ = cfit(ft_,cv_{:});
end

% Plot this fit
h_ = plot(cf_,'fit',0.95);
legend off; % turn off legend from plot method call
set(h_(1),'Color',[1 0 0],...
'LineStyle','-','LineWidth',2,...
'Marker','none','MarkerSize',6);
legh_(end+1) = h_(1);
legt_{end+1} = 'fit 2';

hold off;
legend(ax_,legh_, legt_);

coeffvals = coeffvalues(cf_);
c_a = [
coeffvals(1);coeffvals(4);coeffvals(7);coeffvals(10);coeffvals(13)]
c_b = [
coeffvals(2);coeffvals(5);coeffvals(8);coeffvals(11);coeffvals(14)]
c_c = [
coeffvals(3);coeffvals(6);coeffvals(9);coeffvals(12);coeffvals(15)]
save coeff_a.dat c_a -ascii
save coeff_b.dat c_b -ascii
save coeff_c.dat c_c -ascii

```

```

modelo_fit = formula(cf_)
%coefficienti c=2*sigma^2
sigma_mat = sqrt(c_c./2);
fwhm_mat=2*sqrt(2*log(2))*sigma_mat';

sigma=sprintf('%g \n', sigma_mat)
FWHM = sprintf('%g \n', fwhm_mat)

```

DSP Monochromatic Source

PHA_pileup.m

```

%% PHA with PILE-UP REJECTION
clear all
close all
clc
global k
i=1
pile_up=[];
no_pile_up=[];
while i<5
clear signal
    n_i=int2str(i);
    nfilename=['200nm_20uA_2500frames_' n_i '.dat'];
    signal=load(nfilename);
    for k=1:5
        clear mat mat_def
        as_k=signal(:, (1+(k-1)*500):(k*500));
        mat(:,:)=pulse_detection(as_k,500);
        pile_up=cat(2,pile_up,mat);
        mat_def(:,:)=pile_up_reject(mat(:,:));
        no_pile_up=cat(2,no_pile_up,mat_def);
    end
    i=i+1
clear signal
%%end

%% Once the data has been processed...

mediacon=mean(pile_up');
mediasin=mean(no_pile_up');

t=[1:length(mediacon)]*1e-7;
figure, plot(t,mediacon,t,mediasin)

data=mean(351:length(mediasin));
tdata=t(351:length(t));;

bs_single=mean(pile_up(1:300,:));
mat_bl_con= repmat(bs_single,size(pile_up,1),1);
pile_up_new=pile_up-mat_bl_con;
max_con=max(pile_up(375:425,:));

```

```

bs_single=mean(no_pile_up(1:300,:));
mat_bl_sin= repmat(bs_single,size(no_pile_up,1),1);
no_pile_up_new=no_pile_up-mat_bl_sin;
max_sin=max(no_pile_up(375:425,:));

[xcon,ycon]=histogram(max_con);
[xsin,ysin]=histogram(max_sin);

```

Functions used by PHA_pileup

pulse_detection.m

```

%%PULSE DETECTION ALGORITHM
function [mat]=pulse_detection(as,nfile);

tic
% nfile=500;
% as=X200nm_20uA_2500frames_1(:,1:nfile);

%LOW PASS FILTER 2DESIGN
% Equiripple Lowpass filter designed using the FIRPM function. All
frequency values are in Hz.
Fs = 10000000;           % Sampling Frequency
Fpass = 70000;           % Passband Frequency
Fstop = 110000;          % Stopband Frequency
Dpass = 0.057501127785;  % Passband Ripple
Dstop = 0.001;           % Stopband Attenuation
dens = 20;               % Density Factor
% Calculate the order from the parameters using FIRPMORD.
[N, Fo, Ao, W] = firpmord([Fpass, Fstop]/(Fs/2), [1 0], [Dpass,
Dstop]);
% Calculate the coefficients using the FIRPM function.
b = firpm(N, Fo, Ao, W, {dens}); %Numerator

as_filt=filtfilt(b,1,as);

%length of the box derivative function
n=100;
ceros=zeros(n/2,nfile);

% as_filt=cat(1,ceros,as_filt);
% as_filt=cat(1,as_filt,ceros);
%
% asf=cat(1,ceros,as);
% asf=cat(1,asf,ceros);

asf=as;
boxcar=ones(n,1); %column vector
boxcar(1:n/2)=-1;

```

```

%Baseline restoration samples
if n<300
    nbaseline=400;
else
    nbaseline=ceil(2*n/3);
end

ncamp=1200;

c1=1;
clear res
clear mat mat_filt
for m=1:nfile
    res(:,m)=(1/n)*conv(boxcar,asf(:,m));
    k=1;c2=0;
    for i=1:length(asf)
        if(k<length(asf))
            if ((res(k,m)<=-2e-3)&(c2==0))
                mat_pointer(1,c1)=m-1;
%frame number
                mat_pointer(2,c1)=k;
%time pointer
                if (k>nbaseline)&(k+ncamp<length(asf))
                    mat(:,c1)=asf(k-nbaseline:k+ncamp,m);
%impulses without any filtering
                    mat_filt(:,c1)=as_filt(k-nbaseline:k+ncamp,m);
%impulses LPF
                    c1=c1+1;
                    c2=700;
                    end
                    %k=k+699;
                else k=k+1;
                    if(c2~=0)
                        c2=c2-1;
                    end
                end
            end
        end
    end
end

tsub=(1:ncamp+301)*1e-7;
tres=(1:length(res))*1e-7;
toc

```

pile_up_reject.m

```

function [mat_def]=pileup_reject(mat)

%% Rejection of Pile-Ups
mat_pulse(:,:)=mat(350:950,:);
mat_copy(:,:)=mat(:,:);

area_pulse=trapz(mat_pulse);

```

```

area_mat=trapz(mat_copy);

%INITIAL GUESS (from previous run)
area_media_pulse=2.10;
area_media=2.6;

sigma_0=sqrt(cov(area_mat));
sigma_mat=sigma_0;

delta=1;

for j=1:length(area_pulse)
    ratio(j)=area_pulse(j)/area_mat(j);
end
r_media=2.1/2.6;
%r_media=area_media_pulse/area_media;

while abs(delta)>0.001 | delta==1
    for i=1:length(area_pulse)
        if (ratio(i)-r_media)>0.1*r_media | (abs(area_mat(i)-
area_media)>0.5*sigma_mat)
            mat_copy(:,i)=0;
        end
    end
    indices=find(mat_copy(1,:));
    mat_def=[];
    for i=1:length(indices)
        mat_def(:,i)=mat_copy(:,indices(i));
    end
    mat_pulse=mat_def(350:950,:);

    area_pulse=trapz(mat_pulse);
    area_mat=trapz(mat_def);

    area_media_pulse=mean(area_pulse);
    area_media=mean(area_mat);

    delta=sigma_mat-sqrt(cov(area_pulse));

    sigma_mat=sqrt(cov(area_pulse));

end

% PLOT THE RESULTS
% med_def=mean(mat_def');
% figure, plot(med_def)
% hold all
% media_pileup=mean(mat');
% plot(media_pileup)

```