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A Fundamental Solution Method for Modeling Thin Geological Barriers

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Abstract. *An adaptation of the fundamental solution method (MFS) is developed for potential flow in the presence of barriers. Singularities produced by such objects are expressed analytically through the complex variables techniques, and the resultant singular solution is coupled with the MFS. This can be done by the semianalytical nature of the MFS. The numerical results for sample problems confirm that the proposed model is an inexpensive, accurate and reliable alternative for computing streamline distributions in two dimensional reservoirs.*

1 Introduction

Numerical solution of flow around thin geological barriers is standard topic in reservoir and groundwater modelling [1, 2]. In this context the usual approach is to assign zero values to transmisibilities that intercepts the barriers. Such technique requires very refined grids or good alienation of grid blocks edges with the barriers. In general, both conditions need substantial preprocessing in standard numerical schemes. Therefore, the search of new numerical methods for simulation of thin or slim geological barriers is a current research topic in reservoir engineering [3]. In the last ten years new types of meshfree numerical methods have been reported in the technical literature [4]. In particular, the MFS is a meshfree method without an optimum formulation yet. Its modern development have been reported in [5, 6]. Some applications of MFS to reservoir simulation problems have been published for homogeneous and sectionally homogeneous reservoir [7, 8]. An original adaptation of MFS to the numerical simulation of fluid flow around thin geological barriers in two dimensional reservoirs was developed [9]. This short communication presents some numerical results obtained in that research work for first time. Our article is divided in four sections: barrier and well mathematical models, the new numerical scheme, numerical results, and conclusion.

2 Barrier and well models

Elementary complex variable theory provides standard mathematical models for a thin barriers and wells in two dimensional reservoirs [3, 9, 10]. For simplicity it will be assumed that the barrier is a straight horizontal line, its center in the origin, and its length is 2a. Under these considerations the complex potential associated to a barrier immersed in a constant velocity field V_o is

$$CB(z, a, \alpha, V_o) = -V_o(z\cos(\alpha) - i\sqrt{z^2 - 4a^2}sin(\alpha)) \quad (1)$$

where α is the angle between the barrier and V_o and $z = x + iy$ is a complex variable for position. Any straight barrier can be modeled by a rotation and translation of (1).Although the above model is extremely simple, it can be generalized to thin barriers of arbitrary form [3]. The injection and production wells in a reservoir are represented by the complex potential associated to sources and sinks. This complex potential for a well placed in z_o and the rate q takes the expression

$$CW(z, z_o, q) = \frac{q}{2\pi} \cdot ln(z - z_o) \quad (2)$$

where is the complex logarithm and the rate sign determines a source $(q > 0)$ or a sink $(q < 0)$.

3 New numerical scheme

Fluid flow equation in an homogeneous, single phase, groundwater or oil reservoir Ω is represented by the Poisson equation and a no flow boundary condition on $\partial\Omega$.

$$\Delta P = Sources \text{ in } \Omega \quad (3)$$

$$\frac{\partial P}{\partial \overrightarrow{n}} = 0 \text{ in } \partial\Omega \quad (4)$$

In these expressions P is the reservoir pressure and partial differentiation in (4)is a directional derivative along the external normal vector $\overrightarrow{n}$ to the boundary. The Laplacian Δ is a combination of Darcy's law and continuity equation for incompressible flow. Sources terms in (3) are generalized functions associated to barriers and wells inside the reservoir Ω. The superposition principle and the complex potentials for barriers and wells allow us to write the source terms in (3) as follows

$$Sources = \Delta[Re(\sum_i CW(z, z_i, q_i) + \sum_j T_j(CB(z, a_j, \alpha_j, V_{oj})))] \quad (5)$$

where T_j is an appropriated Moëbius transformation and Re is the real part of a complex variable. The MFS propose an approximated solution for (3) and (4) given by the following expression

$$P(z) = \sum_k \frac{Q_k}{2\pi} \cdot log|z - z_k| + Re(\sum_i CW(z, z_i, q_i) + \sum_j T_j(CB(z, a_j, \alpha_j, V_{oj}))) \quad (6)$$

where the first summation represents a finite linear combination of free Green functions associated to imaginary wells. Their positions z_k are placed in an uniform distribution around and outside the reservoir Ω. Equation (6) is a formal solution of Poisson equation (3) but it does not satisfy boundary condition (4) for arbitrary values of $\{Q_k\}$. The convenient values of imaginary well rates $\{Q_k\}$ are determined by the evaluation of (6) in (4) at collocation points placed along the boundary $\partial\Omega$. For each collocation point a linear equation for $\{Q_k\}$ is obtained, which are collected in a linear system $A \cdot Q = b$ where Q is a vector with the elements of $\{Q_k\}$ as entries. In general, the last linear system is overdeterminated and its solution is obtained in the least square sense by singular valued decomposition method (SVD). Equation (6) with the imaginary well rates computed by the SVD represents an approximated solution for boundary value problem (3) and (4). The accuracy of such solution will depend on the number of imaginary wells and their distance from the reservoir boundary $\partial\Omega$. In our experience, the closer are the imaginary wells from $\partial\Omega$, the better are the approximated solutions. However, imaginary wells placed very near the boundary $\partial\Omega$ produce a very ill posed linear system. The best results are obtained by convenient choice of number of imaginary wells and their distances to $\partial\Omega$. In our numerical results the pressure P given by (6) will be used to obtain streamline distribution in the reservoir.

4 Numerical results

This section presents a very simple reservoir problem to illustrate an application of the new numerical scheme described in previous section. A square shaped reservoir with side length equal to twenty dimensionless units and center the origin will be consider. It will have two wells and one barrier. Well's positions are $(-7, -7)$ and $(7, 7)$, one one of them is an injector and the other is a producer. Its rates are balanced and unitary. The barrier is horizontal and center at the origin. Its independent variables are $V_o = 0.085$, $\alpha = \pi/4$, and $a = 10$. Figure 1 displays this reservoir geometry. In the same figure a sketch of imaginary wells and collocation points are represented by outside white points and stars points on the boundary respectively. In this test imaginary wells were placed at a distance of 0.5 from reservoir boundary. Pressure

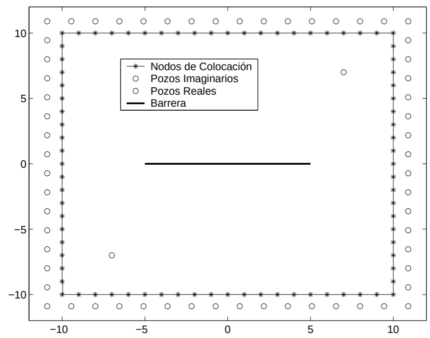

Figure 1: Reservoir geometry with imaginary wells distribution an collocation points.

distribution for the above reservoir configuration can't be easily obtained by analytic methods. Therefore a numerical solution computed on a very refined grid was adopted as analytic solution for validation purposes. Pressure distribution was computed with the new scheme for several number of imaginary wells configurations. The error measured in maximum norm between the analytic and the numerical pressures are presented in table 1. These errors gave evidence that pressure approximation, obtained from equation (6), converges to the analytic solution. Specifically it shows that the number of imaginary wells is the main convergence parameter in the convergence of the new the new scheme. The more imaginary wells in equation (6), more accurate is the approximation obtained by our new scheme. In reservoir engineering and fluid mechanic pressure is an extremely important parameter but it gives little visual insight of fluid

No. Imaginary wells	No. Collocation points	Error
20	40	1.440
100	120	0.050
178	192	0.006

Table 1: Pressure Errors.

flow. Therefore a streamline distribution was obtained from the pressure approximation (6). It is showed in the Figure 2 (left). The imaginary wells and collocation points are presented for reference. Streamlines has a perfect physical correct behavior around the barrier and near de reservoir boundary. Blacks lines in the back of the wells are an artifact of our graphic software and they should not be consider as an error in our approximations. In figure 2 (right) the imaginary wells were removed to show its effect on the streamlines. It shows that their physical behavior is completely wrong away from the barrier and they do not represent a correct solution to our problem. Moreover, a comparison of figures 2 (left) and (right) indicates that imaginary wells are the most important parameter in our new scheme.

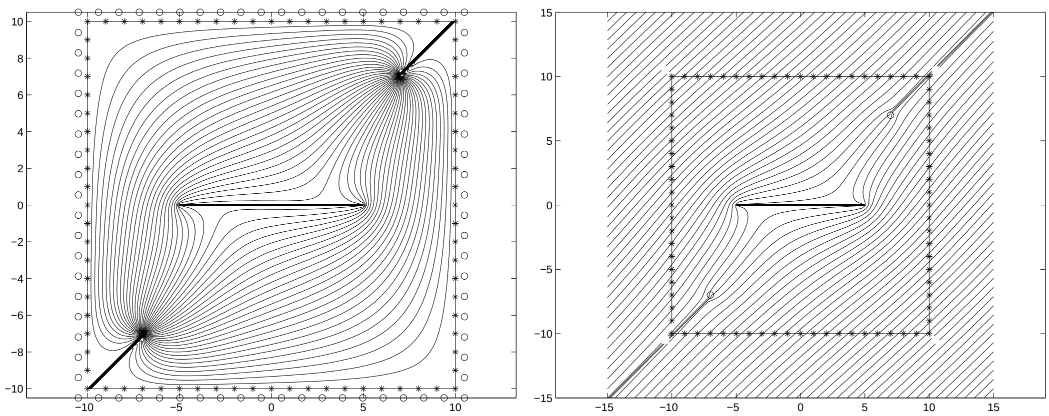

Figure 2: Left display shows the streamlines distribution computed with the new scheme. Right display shows streamlines distribution without imaginary wells.

5 Conclusions

An adaptation of the MFS for the simulation of fluid flow in a two dimensional reservoir with barriers has been presented. A very simple test gave evidence that our new scheme is essentially correct and it produces correct approximations which converges to the real solution. Further results and more complex tests are on the way.

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