# ROBUST MULTIVARIABLE PID DESIGN VIA ITERATIVE LMI

Ernesto Granado, William Colmenares, Omar Pérez

Universidad Simón Bolívar, Departamento de Procesos y Sistemas. Caracas, Venezuela.
e-mail: granado, williamc, operez @usb.ve

## ABSTRACT

This paper presents the design of a robust multivariable PID controller, which guarantees the stability of the closed loop linear systems subjected to polytopic uncertainty. The algorithm is based on an Iterative Linear Matrix Inequality (ILMI) approach. The obtained results may also be used to compute a static output feedback stabilizing controller. The design technique is illustrated with numerical examples.

*Keywords*: PID Control, Robust Control, Uncertainty, Stabilizing Controllers, Linear Matrix Inequality (LMI).

## DISEÑO DE PID MULTIVARIABLE ROBUSTO USANDO LMI ITERATIVAS

### RESUMEN

Este trabajo presenta el diseño de un controlador PID multivariable robusto, el cual garantiza la estabilidad a lazo cerrado de sistemas lineales sujetos a incertidumbre poliédrica. El algoritmo se basa en una Metodología Iterativa de Desigualdades Lineales Matriciales (ILMI). Los resultados obtenidos también pueden ser usados para calcular un controlador estático estabilizante de realimentación de la salida. La técnica de diseño es ilustrada mediante ejemplos numéricos.

*Palabras clave*: Control PID, Control Robusto, Incertidumbre, Controlador Estabilizante, Desigualdades Lineales Matriciales (LMI).

## INTRODUCTION

The PID is the most widespread used industrial controller. In a typical set up, most of the SISO loops are PIDs. It is very popular because of its functional simplicity. Its robust condition has enable the operators to use it, in simple direct ways, obtaining great performances in most of the cases. There exists several tuning methods developed for PIDs, see for instance (Aström & Hägglund, 1995; Corripio, 1996) and the references there in. Most of the strategies are based on an approximate model of the plant (FOPDT or SOPDT). Also, some advance control techniques such as $H_\infty$, MPC and LQG (Grimble, 1991; Katebi & Moradi, 2001; Ge *et al.* 2002; Rusnak, 2000), have been used (in most cases as reduction to the three parameter structure of PIDs).

In this work we present a strategy for calculation robust PID (or PI) multivariable controller for continuous (discrete) linear systems. Much as in (Zheng *et al.* 2002), the approach computes a static output feedback control, applied to an augmented system. The problem is formulated as a Bilinear Matrix Inequality (BMI) and hence an Iterative scheme based on LMIs (ILMI) is proposed to solve the Static Output Feedback (SOF) problem.

Different from (Zheng *et al.* 2000) no additional variables are included, except for a scalar ($\alpha$) and an extension to the discrete and to the robust case (continuous and discrete) is presented. Also, as formulated, the problem (ILMI) always has a solution determine by the parameter $\alpha$, that will be depicted later. Convergence is measured by this parameter, which decrease from iteration to iteration.

No constraints are imposed to the order of the model (such as Zheng *et al.* 2000), nor a particular structure. There is no difference, in our approach, to handle SISO or MIMO systems. Other advance robust methods impose bounds to the order of the model.

The approach is based on a LMI formulation (Boyd *et al.*

1994), and there are available powerful tools ready to solve the problem. In addition, other robust control performance requirements such as: $H_2$, $H_\infty$ and constraints may be easily included as will be featured later.

**PROBLEM STATEMENT**

Consider the continuous or discrete uncertain linear time system described by:

$$\partial[x(t)] = A_I x(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{1}$$

the following PID controller for continuous systems:

$$u(t) = F_1 y(t) + F_2 \int_0^t y(\tau)d\tau + F_3 \frac{dy(t)}{dt} \tag{2}$$

and the following position PI controller for discrete systems:

$$u(t) = F_1 y(t) + F_2 \sum_0^t y(\tau) \tag{3}$$

$x(t) \in \Re^n$, $u(t) \in \Re^l$ and $y(t) \in \Re^m$ are respectively, the state, input and output system, $\partial$ is the derivation operator in continuous time case ( $\partial[x(t)] = \dot{x}(t)$ ) and the delay operator for the discrete time one ( $\partial[x(t)] = x(t+1)$ ). $A_I \in \Re^{n \times n}$, $B \in \Re^{n \times l}$ and $C \in \Re^{m \times n}$ are the system, input and output matrices respectively, and $F_1, F_2, F_3 \in \Re^{l \times m}$ are matrices to be determined and represent the proportional, integral and derivative gain of the controller. Also, matrix $A_I$ is not known precisely. $A_I$ belongs to a polytopic region, defined by:

$$A_I \in \Omega = Co\{A_1, A_2, \cdots A_L\} \tag{4}$$

where $L$ is the number of vertices. $A_I$ may be any matrix obtained by the convex combination of the vertices, i.e.:

$$A_I = \sum_{j=1}^L \alpha_j A_j, \quad \alpha \in \Gamma,$$

$$\Gamma = \left\{ \alpha : \alpha_j \geq 0, \sum_{j=1}^L \alpha_j = 1 \right\} \tag{5}$$

Following the procedure in (Cao *et al.* 1998 and Zheng *et al.* 2002), the closed loop system (1) and (2) [or (3)], may be equivalently represented by an augmented system with static

output feedback of the form:

$$\partial[z(t)] = \bar{A}_I z(t) + \bar{B} u(t), \quad \bar{y}(t) = \bar{C}_I z(t) \tag{6}$$

$$u(t) = \bar{F}\bar{y}(t)$$

where:

$$\bar{A}_I = \begin{bmatrix} A_I & 0 \\ C & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{C}_I = \begin{bmatrix} C & 0 \\ 0 & I \\ CA_I & 0 \end{bmatrix},$$

$$z(t) = \begin{bmatrix} x(t) & \int_0^t y(\tau)d\tau \end{bmatrix}^T \tag{7}$$

for continuous systems, and

$$\bar{A}_I = \begin{bmatrix} A_I & 0 \\ CA_I & I \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ CB \end{bmatrix}, \bar{C}_I = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix},$$

$$z(t) = \begin{bmatrix} x(t) & \sum_0^t y(\tau) \end{bmatrix}^T \tag{8}$$

for discrete systems.

The gains are:

$$\bar{F} = [\bar{F}_1 \quad \bar{F}_2 \quad \bar{F}_3], \quad \bar{F}_1 = (I - F_3 CB)^{-1} F_1,$$
$$\bar{F}_2 = (I - F_3 CB)^{-1} F_2, \quad \bar{F}_3 = (I - F_3 CB)^{-1} F_3 \tag{9}$$

for continuous systems, and

$$\bar{F} = [F_1 \quad F_2] \tag{10}$$

for discrete systems.

$\bar{A}_I$ and $\bar{C}_I$ are the corresponding uncertainty matrices (in the discrete case, $\bar{C}_I$ is indeed certain).

The design objective reduces to finding a matrix $\bar{F}$, such that when the loop is closed in (6):

$$\partial[z(t)] = (\bar{A}_I + \bar{B}\bar{F}\bar{C}_I) z(t) \tag{11}$$

be asymptotically stable.

For continuous systems, once the composite matrix $\overline{F} = [\overline{F}_1 \quad \overline{F}_2 \quad \overline{F}_3]$ is found, the original PID gains can be recovered from:

$$F_3 = \overline{F}_3(I + CB\overline{F}_3)^{-1}, \quad F_2 = (I - F_3CB)\overline{F}_2,$$
$$F_1 = (I - F_3CB)\overline{F}_1 \tag{12}$$

The invertibility of matrix $I + CB\overline{F}_3$ will be discussed later.

**Preliminaries**

Now suppose system (6) is autonomous (i.e. $u = 0$), the condition for quadratic stability is given by:

Lemma 1. (Bernussou, *et al.* 1989). Autonomous continuous system (6) is quadratically stable if and only if there exist symmetric matrix such that:

$$SA_j + SA_j^T < 0, \ \forall \ j = 1,2,\Lambda \ ,L \tag{13}$$

and (Garcia & Bernussou, 1995)

$$A_j SA_j^T - S < 0, \ \forall \ j = 1,2,\Lambda \ ,L \tag{14}$$

for discrete systems.

Finally, let us formulate the condition for quadratic stabilizability for the static output feedback case:

Theorem 1. System (11) is quadratically stable (i.e., there exists a matrix $\overline{F}$, static output gain, that stabilizes all system represented in (6) ) if there exist a matrix $S = S^T > 0$ and a gain $\overline{F}$ such that:

$$\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}_j\right)S + S\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}_j\right)^T < 0,$$
$$\forall \ j = 1,2,\Lambda \ ,L \tag{15}$$

$$\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}\right)S\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}\right)^T - S < 0,$$
$$\forall \ j = 1,2,\Lambda \ ,L \tag{16}$$

for discrete systems.

In this work we follow a similar strategy to that proposed by (Cao, *et al.*, 1998) and introduce an extra term $\alpha$ in the condition (15) [or (16)].

Instead of (15) [or (16)], we will work with:

$$\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}_j\right)S + S\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}_j\right)^T - \alpha S < 0$$
$$\forall \ j = 1,2,\Lambda \ ,L \tag{17}$$

and

$$\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}\right)S\left(\overline{A}_j + \overline{B}\,\overline{F}\,\overline{C}\right)^T - S - \alpha S < 0,$$
$$\forall \ j = 1,2,\Lambda \ ,L \tag{18}$$

where $\alpha$ is a scalar variable added to the problem to assure always a solution to the inequalities.

If $S > 0$ is fixed then (17) [or (18)] is convex in $\alpha$ and $\overline{F}$, similarly, if $\overline{F}$ and $\alpha$ are fixed then it is convex in $S$, in both cases it is easy to compute $\alpha^* = \min \alpha$. If in one of those two problems $\alpha^* \leq 0$, then $\overline{F}$ is a static quadratic stabilizing output gain.

Remark 1. The continuous system (17) guarantees that the continuous system is $\alpha/2$ stabilizable via static output feedback and $\overline{F}$ place the closed loop poles to the left of a vertical line $\text{Re}(s) = \alpha/2$ in the complex plane, when $\alpha^* \leq 0$. Also solution to (18) guarantees the closed loop discrete system locates all its poles within a circle centered in zero with radius $\sqrt{1+\alpha}$.

The following iterative linear matrix inequality (ILMI) algorithm is proposed to solve the problem:

**Algorithm**

For the triplet $(\overline{A}_j, \overline{B}, \overline{C}_j), \forall \ j = 1,2,\Lambda \ ,L$, the algorithm is the following:

Step 1: to initiate the algorithm, we find matrix $S > 0$ and matrix $R$ solutions to the full state feedback quadratic stabilizability ($u(t) = Kz(t)$ ):

$$S\overline{A}_j^T + \overline{A}_j S + R^T \overline{B}^T + \overline{B}R < 0, \quad S > 0$$
$$\forall \ j = 1,2,\Lambda \ ,L \tag{19}$$

for continuous systems, and

$$\begin{pmatrix} S & \overline{A}S + \overline{B}R \\ S\overline{A}^T + R^T\overline{B}^T & S \end{pmatrix} > 0,$$

$$\forall \; j = 1,2,\cdots,L \tag{20}$$

for discrete systems, where $R = KS$.

Step 2: with $S > 0$ obtained from step 1, compute $\alpha^*$ ($\alpha^* = \min \alpha$) and $\overline{F}$ subject to:

$$S(\overline{A}_j + \overline{B}\overline{F}\overline{C}_j)^T + (\overline{A}_j + \overline{B}\overline{F}\overline{C}_j)S - \alpha S < 0$$

$$\forall \; j = 1,2,\Lambda\,,L \tag{21}$$

for continuous systems, and

$$\begin{pmatrix} S + \alpha S & \overline{A}_j S + \overline{B}\overline{F}\overline{C}S \\ S\overline{A}_j^T + S\overline{C}^T\overline{F}^T\overline{B}^T & S \end{pmatrix} > 0$$

$$\forall \; j = 1,2,\cdots,L \tag{22}$$

for discrete systems.

If $\alpha^* \le 0$ stop, $\overline{F}$ is a quadratic stabilizing static output feedback gain; else go to step 3.

Step 3: fix $\overline{F}$ (obtained from step 2), look for $\alpha^*$, by starting a search process in a scalar variable ($\alpha$), computing $S > 0$ that solve (21) [or (22)] then go to step 2.

The algorithm continues until an $\alpha^* \le 0$ is found (an hence a solution controller obtained) or until $\alpha^*$ converges to some positive value ($\alpha_i^* - \alpha_{i-1}^* \approx 0$), in which case there may be a solution but the algorithm fails to find it.

Remark 2. It has to be remarked that LMI (19) [or (20)] is a necessary condition for the existence of a static stabilizing gain, that is, if the algorithm fails to start because there is no solution to LMI (19) [or (20)], then there is no static stabilizing gain, and then, the original system (1) can't be stabilized by a PID controller.

Remark 3. The $\alpha^*$ obtained in one iteration (step 2 or step 3) is an upper bound to the one computed in the following (step 3 or step 2), since what we are really looking for is an

$\alpha^*$ zero or negative, the search in step 3 may start in a value smaller than the previous $\alpha^*$.

Remark 4. Once $\alpha^* \le 0$ is reached, the associated solutions ($\overline{F}$) is not necessarily a good one (poor margins, etc) and hence the quest process (between steps 2 and 3) may continue until a satisfactory solution is obtained. Recall that in our case $\alpha^*$ is a measure of performance and it decreases from iteration to iteration. Let us go back to the computation of the PID parameters.

Proposition 1. (Zheng *et al.* 2002). Matrix $I - F_3CB$ is invertible if and only if matrix $I + CB\overline{F}_3$ is invertible, where $F_3$ and $\overline{F}_3$ are related to each other by:

$$\overline{F}_3 = (I - F_3CB)^{-1}F_3 \text{ or } F_3 = \overline{F}_3(I + CB\overline{F}_3)^{-1} \tag{23}$$

**Proof**. in (Zheng *et al.* 2002).

There are two approaches to deal with Proposition 1 in the design of the feedback matrices (Zheng *et al.* 2002). The first approach is to do nothing but post-check whether $I + CB\overline{F}_3$ and hence $I - F_3CB$ are invertible. This is based on the observation that the probability of finding $\overline{F}_3$, which makes $I + CB\overline{F}_3$ singular is zero in the whole possible parameter space consisting of $\overline{F}_3$. The second approach is to add the following conservative LMI:

$$I + (CB\overline{F}_3)^T + CB\overline{F}_3 > 0 \tag{24}$$

In our case, we always used the first approach.

**NUMERICAL EXAMPLES**

In this section, we present three examples that illustrate the implementation of the proposed tuning controller algorithm. For these examples, the software LMI control toolbox was used to compute the solution.

**Example 1**

First, feature an application to multivariable continuous systems. The model is a «benchmark» problem taken from the literature and used in many works. The system is composed of two masses and a spring like is shown in Figure 1.
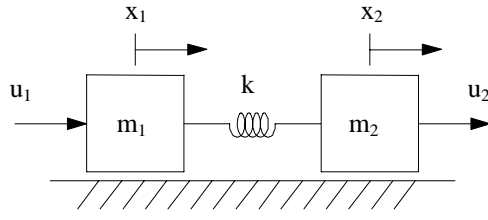
**Figure 1.** System

The system model is represented by (1), where:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k}{m_1} & \dfrac{k}{m_1} & 0 & 0 \\ \dfrac{k}{m_2} & -\dfrac{k}{m_2} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{m_1} & 0 \\ 0 & \dfrac{1}{m_2} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where $m_1$ and $m_2$ are the two masses and $k$ (uncertain parameter) is the constant of stiffness of the spring. The state variables $x_1, x_2$ represent the position of the two masses, while $x_3, x_4$ are speeds.

The methodology is applied by considering the followings values $m_1 = m_2 = 0.1$. We will suppose that the range for the uncertainty parameter is:

$$0.1 \le k \le 2$$

Solving the problem we obtain an $\alpha = -0.0102$ and the following matrix of the controller parameters.

$$F_1 = \begin{bmatrix} -4.8433x10^6 & 4.2211x10^6 \\ 4.2211x10^6 & -4.8433x10^6 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -2.4187x10^5 & -1.9477x10^5 \\ -1.9477x10^5 & -2.4187x10^5 \end{bmatrix},$$

$$F_3 = \begin{bmatrix} -8.8221x10^5 & 1.1294x10^5 \\ 1.1294x10^5 & -8.8221x10^5 \end{bmatrix}$$

Figure 2 show the output time profile when the initial condition is $x(0) = [2,1,0,0]$ and the uncertain parameter $k$ ranges from 0.1 to 2.
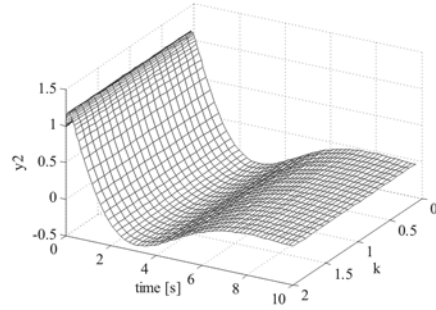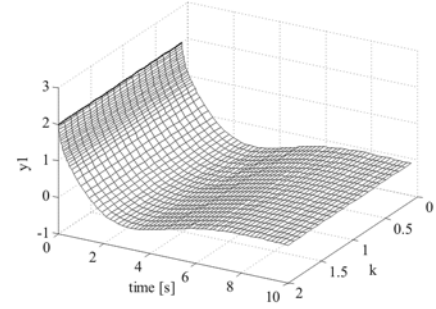




**Figure 2.** Output time profile ( $0.1 \le k \le 2$ ).

**Example 2**

In this example, we feature the algorithm for a multivariable discrete system. The model is taken from (Maciejowski, 2002) (slightly modified: a uncertain parameters $k$ is added to the system matrix). The system is a paper-making machine and the discrete model with sampling period of 2 min, is given by:

$$A = \begin{bmatrix} 0.0211 & 0 & 0 & 0 \\ 0.1062 & 0.4266k & 0 & 0 \\ 0 & 0 & 0.2837 & 0 \\ 0.1012 & -0.6688 & 0.2893 & 0.4266k \end{bmatrix},$$

$$B = \begin{bmatrix} 0.6462 & 0.6462 \\ 0.2800 & 0.2800 \\ 1.5237 & -0.7391 \\ 0.9929 & 0.1507 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We will suppose that the ranges for the uncertainty parameters are:

$$0.5 \le k \le 2$$

Solving the problem we obtain an $\alpha = -0.0902$ and the following matrix of the controller parameters.

$$F_1 = \begin{bmatrix} 0.3427 & -0.2997 \\ -1.1610 & 0.0373 \end{bmatrix}, \ F_2 = \begin{bmatrix} -0.1138 & -0.2516 \\ -0.5120 & -0.0195 \end{bmatrix}$$

Figure 3 show the output time profile when the initial condition is $x(0) = [0,1,0,1]$ and the uncertain parameter $k$ ranges from 0.5 to 2.
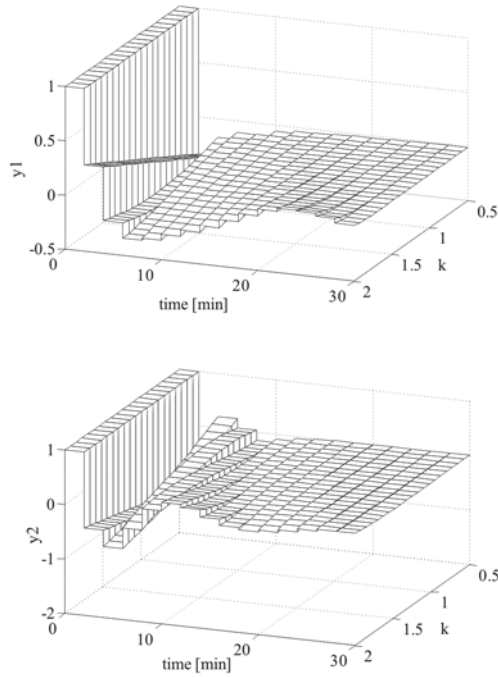




**Figure 3.** Output time profile ($0.5 \le k \le 2$).

**Example 3**

In this example, we use the algorithm to tune a continuous SISO PID. The approach is compared with other tuning techniques such as: ZN (Ziegler & Nichols, 1942) and SIMC (Skogestad, 2001). Note that $L = 1$, i.e., there is no uncertainty.

The model, taken from (Skogestad, 2001), is:

$$G(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$$

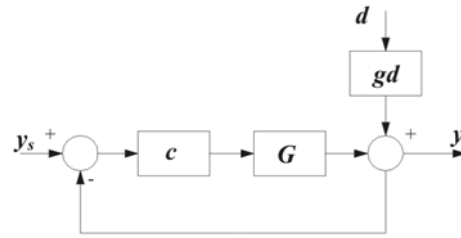Figure 4 show the block diagram of the feedback control system.



**Figure 4.** Block diagram of the feedback control system. We consider an input («load») disturbance ($gd=G$).

Solving the problem we obtain an $\alpha = -10.2388$ and the following PID controller parameters:

$$F_1 = 43.2219, \ F_2 = 115.1623, \ F_3 = 4.4085.$$

Figure 5 shows the output time profile for the different PID tuning techniques. The values of the PID parameters are taken from table 4 of (Skogestad, 2001) and showed in Table 1.
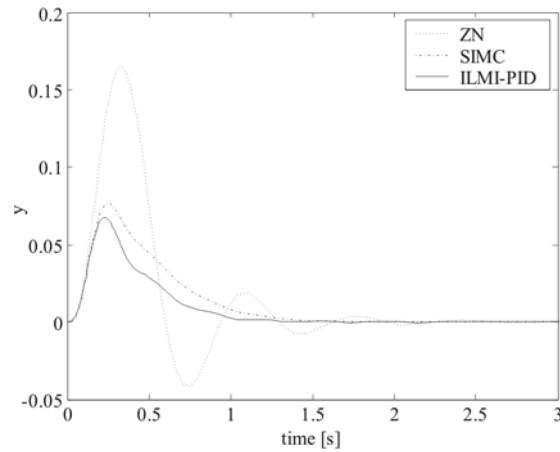


**Figure 5.** Output time profile with different PID's; setpoint equal to zero and load disturbance of magnitude 3 at t=0.

**Table 1.** PID settings for different tuning techniques.

|          | $F_1$    | $F_2$    | $F_3$  |
|----------|----------|----------|--------|
| ZN       | 18.2     | 65       | 1.274  |
| SIMC     | 35.48    | 79.91    | 3.938  |
| ILMI-PID | 43.2219  | 115.1623 | 4.4085 |

## OTHER PERFORMANCE SPECIFICATIONS

In this section, we included other performance requirements for the sake of enrichment of the proposed strategy and to illustrate the procedure. In particular, we will only feature the case of $H_\infty$ norm for discrete systems. Applications to $H_2$ norm or $H_2$, $H_\infty$ norm of continuous systems is straight forward.

Consider the discrete system:

$$x(t+1) = A_I x(t) + Bu(t) + B_1 w(t)$$
$$y(t) = Cx(t) \tag{25}$$

where $w(t)$ is an external disturbance and, it is desired additionally that the discrete PI controller (3) assure the $H_\infty$ norm from $w(t)$ to $y(t)$ is less than (a given scalar value).

When the extra state variable $\sum_0^t y(\tau)$ is included, the augmented system results:

$$z(t+1) = \overline{A}_I z(t) + \overline{B}u(t) + \overline{B}_1 w(t)$$
$$\overline{y}(t) = \overline{C}_I z(t)$$
$$\xi(t) = \overline{C}_\infty z(t) \tag{26}$$
$$u(t) = \overline{F}\overline{y}(t)$$

where $\overline{A}_I, \overline{B}, \overline{C}_I$ are as defined in (8),

$$\overline{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad \overline{C}_\infty = \begin{bmatrix} C & 0 \end{bmatrix} \tag{27}$$

The problem is to find a stabilizing static output feedback gain $\overline{F}$ such that the transfer function $T_{wy} = \overline{C}_\infty \left( \delta I - \overline{A}_I \right)^{-1} \overline{B}_1$ satisfy:

$$\left\| T_{w\xi} \right\|_\infty < \gamma \tag{28}$$

It is well known that condition (28) is satisfied if and only if there exists a matrix $S > 0$ and a gain $\overline{F}$ such that:

$$\begin{bmatrix} S & \overline{A}_j S + \overline{B}\,\overline{F}\,\overline{C}S & \overline{B}_1 & 0 \\ \overline{A}_j^T + S\overline{C}^T \overline{F}^T \overline{B}^T & S & 0 & S\overline{C}_\infty^T \\ \overline{B}_1^T & 0 & \gamma I & 0 \\ 0 & \overline{C}_\infty S & 0 & \gamma I \end{bmatrix} > 0 \tag{29}$$
$$\forall \ j = 1,2,\cdots,L$$

Therefore, to compute a PI discrete controller with a $H_\infty$ specification, it suffices to change condition (22) for:

$$\begin{bmatrix} S + \alpha S & \overline{A}_j S + \overline{B}\,\overline{F}\,\overline{C}S & \overline{B}_1 & 0 \\ \overline{A}_j^T + S\overline{C}^T \overline{F}^T \overline{B}^T & S & 0 & S\overline{C}_\infty^T \\ \overline{B}_1^T & 0 & \gamma I & 0 \\ 0 & \overline{C}_\infty S & 0 & \gamma I \end{bmatrix} > 0 \tag{30}$$
$$\forall \ j = 1,2,\cdots,L$$

And continue.

## CONCLUSIONS

In this work, an effective algorithm based on LMIs that enables the computation of the parameters of a robust PID multivariable controller for continuous and PI for discrete systems, was presented.

Sufficient conditions are established to guarantee the quadratic stability of the closed loop of the uncertain systems with polytopic uncertainty.

Neither specific requirement in the system structure, nor in its order is imposed to apply the methodology.

The paper is an extension to robust PI and PID controller computation for continuous and discrete systems.

The algorithm is formulated so that when completion of the iterative process is achieved, not only a stabilizing PI (or PID) controller will be obtained but also a measure of performance, as determined by the parameter $\alpha^*$ and its direct relation to the location of the poles. Also, as featured, the algorithm might easily incorporate other specifications such as $H_2$ and $H_\infty$.

When applied to SISO loops without uncertainty, the approach yield comparable results as those of well known IMC-PID techniques.

The result may be used to compute a static output feedback stabilizable controller.

## REFERENCES

Aström, K. J. & Hägglund, T. A. (1995). PID controllers: theory design and tuning. Instrument Society of America, Research Triangle Park, NC, USA.

Bernussou, J., Peres, P. L. D. & Geromel, J. C. (1989). A linear programming oriented procedure for quadratic

stabilization of uncertain system. Systems and Control Letters 13, 65-72.

BOYD, S., EL GHAOUI, L., FERON, E. & BALAKRISHNAN, V. (1994). Linear Matrix Inequalities in System and Control Theory, vol. 15, SIAM, Philadelphia.

CAO, Y.-Y., LAM, J. & SUN, Y.-X. (1998). Static Output Feedback Stabilization: An ILMI Approach. Automatica 34, 1641-1645.

CORRIPIO, A. B. (1996). Tuning of industrial control systems, Instrument Society of America, Research Triangle Park, NC, USA.

GARCIA, G. & BERNUSSOU, J. (1995). Pole Assignment for Uncertain Systems in a Specified Disk by State Feedback. IEEE Transactions on Automatic Control 40, 184-189.

GE, M., CHIU, M-S. & WANG, Q-G. (2002). Robust PID controller design via LMI approach. Journal of Process Control 12, 3-13.

GRIMBLE, M. J. (1991). Hinf PID controllers, Trans. IMC 13 (5), 112-120.

KATEBI, M. R. & MORADI, M. H. (2001). Predictive PID controllers. IEE Proc.-Control Theory Appl. 148 (6), 478-487.

MACIEJOWSKI, J. (2002). Predictive Control: with constraints. Pearson Education Limited, London.

RUSNAK, I. (2000). The generalized PID controller and its application to control of ultrasonic and electric motors. Proc. of IFAC work-shop PID'00, Spain, 125-130.

SKOGESTAD, S. (2001). Simple analytic rules for model reduction and PID controller tuning. Journal of Process Control 13, 291-309.

ZHENG, F., WANG, Q.-G. & LEE, T. H. (2002). On the design of multivariable PID controllers via LMI approach. Automatica 38, 517-526.

ZIEGLER, J. G. & NICHOLS, N. B. (1942). Optimum settings for automatic controller. Tras. Of the A.S.M.E., 64, 759-768.