CAN WE ACCURATELY USE LABORATORY-SCALE RHEOLOGICAL PARAMETERS TO MODEL FIELD-SCALE DEBRIS FLOW?

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ABSTRACT

Numerical models are often used to predict debris flow behavior at field scale. However, rheological parameters that govern fluid behavior are difficult to obtain using conventional rheometers. Generally, laboratory experiments are combined with mathematical modeling to calibrate rheological models. Then, the laboratory-calibrated numerical model is used to predict debris flow behavior at field scales. This paper shows that extrapolating results from laboratory experiments to field scales may lead to non-accurate predictions. Usually, laboratory flumes are just a few meters long, while an actual debris flow event in the field can involve scales of several kilometers. Therefore, a rheological model that provides good results in the laboratory, does not necessarily replicate a field event. In this paper, we analyzed field-scale predictions using numerical models whose rheological parameters are adjusted from laboratory experiments. Results from water flow in large scale experimental facilities were simulated by different rheological models. Rheological parameters were adjusted using a mathematical model. Results showed that adjusting specific parameters, all rheological models could be adequately calibrated when using laboratory data for different flow conditions, even though water was the fluid used in laboratory experiences. However, when models were applied to a field event, there were significant differences in their predictions. This suggests that adjusting rheological parameters by using laboratory scale experiments may not be applicable for field-scale debris flow simulations.

Keywords: Debris flows, Resistance laws, Rheological behavior of mixtures, Numerical simulation, Channel slope.

¿ES VÁLIDO UTILIZAR CON PRECISIÓN PARÁMETROS REOLÓGICOS OBTENIDOS EN LABORATORIO PARA MODELAR ALUDES TORRENCIALES A ESCALA REAL?

RESUMEN

Para predecir el comportamiento de aludes torrenciales a escala real se utilizan modelos numéricos. Sin embargo, los parámetros reológicos que permiten representar los fluidos que componen el flujo son difíciles de obtener utilizando reómetros convencionales. En general experiencias de laboratorio son combinadas con modelos matemáticos para calibrar los modelos reológicos. Luego, el modelo numérico calibrado es utilizado para predecir el comportamiento de los aludes torrenciales en campo. Sin embargo, extrapolar resultados a partir de experimentos de laboratorio a escala real puede conducir a predicciones no precisas. Normalmente, los canales de laboratorio se construyen de tamaño reducido a pocos metros de largo mientras que un alud torrencial puede desarrollarse en escala de kilómetros. En consecuencia, un modelo reológico que permite obtener buenos resultados a escala de laboratorio puede no ser suficientemente preciso a escala real. En este artículo se analizan predicciones a escala de campo utilizando modelos numéricos ajustados a partir de experiencias de laboratorio. Resultados obtenidos para flujo de agua en experiencias de laboratorio a gran escala fueron modeladas utilizando modelos reológicos distintos. Los parámetros de los modelos reológicos se ajustaron utilizando un modelo matemático. Los resultados muestran que aún cuando el fluido considerado en las experiencias de laboratorio fue el agua, todos los modelos reológicos pudieron ser adecuadamente calibrados. Sin embargo, cuando los modelos fueron aplicados a un evento a escala real, las predicciones obtenidas resultaron completamente distintas. Esto sugiere que ajustar modelos reológicos utilizando experimentos a escala de laboratorio para luego utilizarlos en predicciones de eventos a escala real podría ser una práctica muy cuestionable.

Palabras clave: Alud torrencial, Leyes de resistencia, Comportamiento reológico de mezcla, Simulación numérica, Pendientes de canales.

INTRODUCTION

Estimation of debris flow behavior is a key topic in hazard assessment in mountainous areas. In particular, the identification of an appropriate rheological model has long been regarded as the most important aspect to the successful interpretation, modeling, and prediction of debris flow behavior. A great deal of work has been done in order to establish the most suitable rheological formula that models debris flow behavior in all different phases.

Thus, a variation of a Newtonian model has been employed by Takahashi (2000). Newtonian turbulent model used in water flow channel modeling, based on the introduction of the Manning friction coefficient, has been used successfully. Some authors such as Armanini *et al*. (2003) employ different methods that estimate rheological parameters in a Bingham Plastic model. The popular Herschel-Bulkley model has been used by a significant number of authors such as Coussot (1992).

More complex models such as the Generalized Viscoplastic Fluid (GVP) have been employed by Chen (1988). The dilatant inertial model has been employed in actual events by Lo & Chau (2003) and, in laboratory tests, by Ghilardi *et al*. (2003). One of the most popular models was proposed by O'Brien *et al*. (1993) and it constitutes the base line for the popular FLO2D model. Voellmy type fluids have been used with success by McArdell *et al*. (2003).

Therefore, it is necessary to adjust a number of parameters depending on which model is chosen for simulating debris flow. In particular, there are several studies that have looked for the best adapted models to numerical debris flow simulation. Rickenmann & Koch (1997) tested five different rheological models, while McArdell *et al*. (2003) used six different rheological models with numerical experiments. In both studies one or two rheological models were able to reproduce field events in a satisfactory way. Model parameters were adjusted in order to reproduce physical characteristics of the debris flow motion, such as velocity or highest deep at some test section. In this way, the best formula is chosen to simulate debris flow motion in particular areas.

On the other hand, some authors such as Iverson (2003) argued that non-unique rheology is likely to describe the range of mechanical behavior exhibited in debris flow. This claim is based on quantitative data obtained from large-scale experiment in a flume of around 66m long.

In practice, rheological model should be selected in advance, in order to establish several scenarios of disaster prevention. To do this, rheology could be established from laboratory experiments. However, laboratory scale may not be adequate for reproducing field behavior of debris flow. The range of shear rate and shear stresses considered in a laboratory experiment, including large scale experiments, is far from those achieved in real events.

In this study, we use a 1D model developed by Rodríguez *et al*. (2006), that replicates debris flow. The aim is to compare numerical results employing four different constitutive equations. This numerical model is a high resolution and non-oscillatory scheme, based on the finite volume method. Parameters of different rheological models were adjusted by using large scale WES experiments (1960) for two different bed channel roughness. WES experiments have been widely used for calibrating numerical models because they are among the experiments carried out at the largest scales. Once rheological models were adjusted, we considered two different hypothetical cases of dam-break problems, with the same roughness used at the WES tests. Therefore, comparisons are made between predictions of flow depth in several test stations as a function of time. Finally, some conclusions about the possibility of determining rheological parameters from laboratory experiments and to predict debris flow behavior in actual events are provided.

GOVERNING EQUATIONS

The mathematical model employed for the simulation assumes a 1D homogeneous water-sediment current over a rigid bed in unsteady conditions as in Arattano & Franzi (2003). It also assumed that no sediment deposition or aggradation occurs along the torrent, and that there is no momentum exchange between the water-sediment mixture and the bed. So, equations for flow in a 1D channel with rectangular cross section b and with bed slope θ are:

$$\frac{\partial h}{\partial t} + \frac{1}{b}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + gA\,\frac{\partial h}{\partial x}\cos\theta + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gAS_f - gAS_0 = 0 \tag{2}$$

where:

x is the spatial coordinate measured along the length of the channel, t is the time, h is the flow depth, $A(x,t)$ the area of the flow cross section, $Q(x,t)$ the water-sediment discharge, g the gravity acceleration, S_0 the bed slope ($\tan\theta$) and S_f the friction loss.

FLOW RESISTANCE LAWS

For taking into account different kinds of rheological models, the friction term in the equation of motion is modified (Rodríguez *et al*. (2006) for details). Thus, the friction term was modeled by using four different flow resistance laws in the same way as Rickenmann & Koch (1997). So, Newtonian turbulent, Bingham laminar, Voellmy and Dilatant inertial fluid models were considered.

Homogeneous single phase models

For a Newtonian turbulent model S_f is obtained from:

$$S_f = \frac{n^2 V|V|}{R_h^{4/3}} \quad (3)$$

where:

n is the Manning's roughness coefficient and $R_h=A/P$ is hydraulic radius and P being the wetted perimeter.

For the Bingham plastic model:

$$S_f = \frac{\tau_0}{\rho g h} \quad (4)$$

where:

τ_0 is the basal shear stress obtained as the solution of the cubic equation (Rickenmann & Koch, 1997)

$$2\tau_0^3 - 3\left(\tau_B + 2\frac{\mu_B Q}{h^2}\right)\tau_0^2 + \tau_B^3 = 0 \quad (5)$$

where:

τ_B is the Bingham yield stress and μ_B the Bingham viscosity.

Solid-liquid mixtures models

For a dilatant inertial fluid (Rickenmann & Koch, 1997):

$$S_f = \frac{V|V|}{h^3 \xi^2} \quad (6)$$

where:

ξ is a parameter accounting for grain and concentration properties in granular flows.

In the case of a Voellmy fluid (Rickenmann & Koch, 1997):

$$S_f = \frac{V|V|}{hC^2} + \cos\theta\tan\delta \quad (7)$$

where:

C and δ are the Chézy roughness coefficient and the internal friction angle, respectively.

RHEOLOGICAL PARAMETERS ESTIMATION

In all cases, the rheological parameters were obtained by comparison with the results of two dam failure experiments made at the Waterways Experiment Station (WES), U.S. Corps of Engineers (1960). The fluid used in the experiments was water and the bed downstream of the dam site was initially dry.

In these experiments, it was used a rectangular wooden flume, lined with plastic-coated plywood, 122m long and 1.22m wide with a bottom slope of S_0=0.005. The model dam was placed at the midsection (x=61m), impounding water to a depth of 0.305m. The two experiments differed only in the value of hydraulic resistance on the flume bottom: test 1.1 refers to a smooth bottom (n=0.009) while test 1.2 corresponds to a rough one (n=0.050).

Table 1 shows the values of the different coefficients used for each rheological model for both types of bed.

Table 1. Parameters estimated for each rheological model.

Model	Parameter	Smooth bed	Rough bed
Manning	N	0.009	0.05
Voellmy	C^2	70.71 [$m^{1/2}s^{-1}$]	12.247 [$m^{1/2}s^{-1}$]
	δ	0 [º]	0 [º]
Inercial-Dilatant	ξ	850 [$m^{1/2}s^{-1}$]	100 [$m^{1/2}s^{-1}$]
Bingham	τ_B	0.001 [Pa]	0.01 [Pa]
	μ_B	0.09 [Pa-s]	1.45 [Pa-s]

In Figure 1, it is observed the depth hygrograph predictions of each model for x =-30.5m (upstream dam station), x = 0 (original dam station), x = 24.4m and x = 45.7m (downstream dam stations) for smooth bottom test.

In all test cases, the predictions and the experimental data coincide fairly well. In order to quantify deviation among predictions from rheological models, the correlation index among different predictions was estimated. Table 2 shows the correlation index for WES Test 1.1.

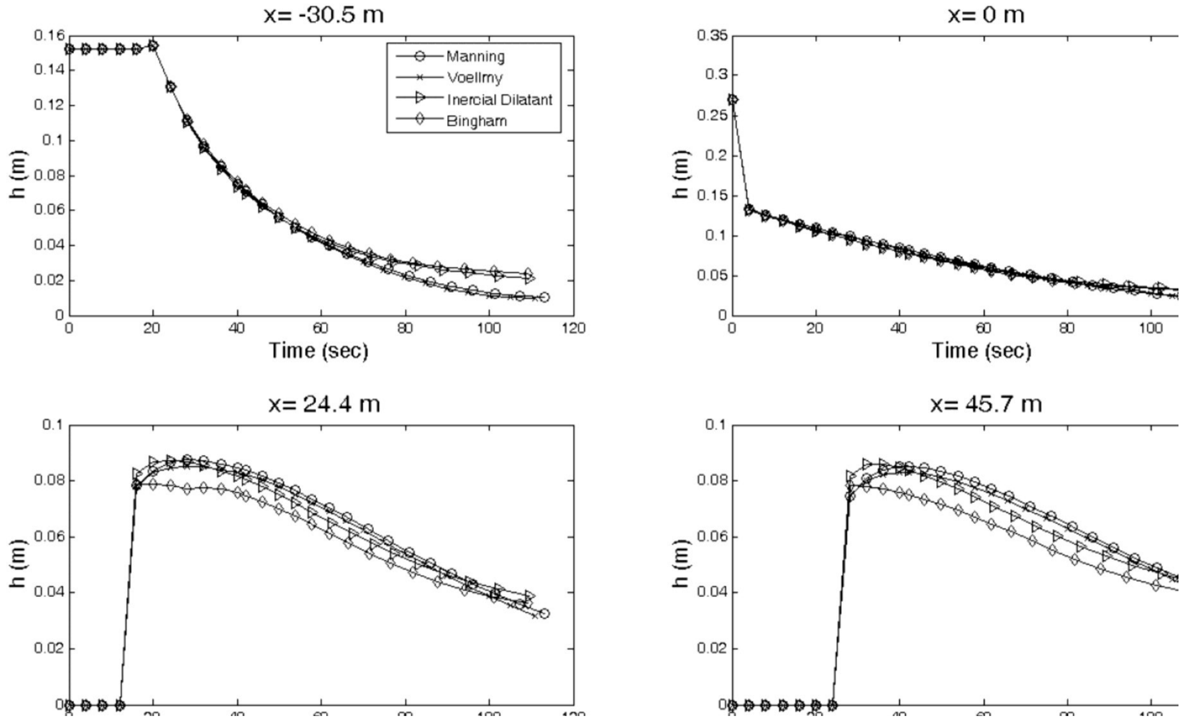

Figure 1. Evolution of computed depths at x=-30.5m, 0m, 24.4m and 45.7m for WES test 1.1.

Table 2. Correlation index among depth predictions for the rheological models.

Station 1 (x=-30.5)

	Manning	Voellmy	Inercial	Bingham
Manning	1.0000	0.9996	0.9863	0.9820
Voellmy		1.0000	0.9844	0.9938
Inercial			1.0000	0.9989
Bingham				1.0000

Station 2 (x=0.0)

	Manning	Voellmy	Inercial	Bingham
Manning	1.0000	0.9986	0.9835	0.9826
Voellmy		1.0000	0.9776	0.9784
Inercial			1.0000	0.9987
Bingham				1.0000

Station 3 (x=24.0)

	Manning	Voellmy	Inercial	Bingham
Manning	1.0000	0.9969	0.9841	0.8737
Voellmy		1.0000	0.9826	0.8863
Inercial			1.0000	0.8937
Bingham				1.0000

Station 4 (x=45.7)

	Manning	Voellmy	Inercial	Bingham
Manning	1.0000	0.9611	0.9434	0.8323
Voellmy		1.0000	0.9884	0.8901
Inercial			1.0000	0.9076
Bingham				1.0000

In all test cases, the predictions and the experimental data coincide fairly well. The greatest differences were found at the last measurement station. This may be due to end effects because this station is the closest one to the flume exit. However, the worse correlation index, 0.83, between Newtonian model and Bingham Plastic model is not very far from 1, and qualitative agreement among all predictions is very good.

Similar trends were observed for the rough bed channel experience. Figure 2 shows the depth hygrographs predictions for models in each measurement station.

Again, the predictions among different models coincide fairly well. The greatest differences were found at downstream measurement stations, but qualitative agreement among all predictions is very good and the lower correlation index, 0.82, in this case, between Voellmy model and Bingham Plastic model, has the same order of those found in the smooth bed case. As a consequence n=0.82 will be used as reference value for establishing similarities among predictions in different models.

We concluded that, for practical purposes, these rheological parameters provide predictions of the fluid flow characteristics (depth flow, front velocities, etc.) that can be considered equals, independently of the rheological model chosen.

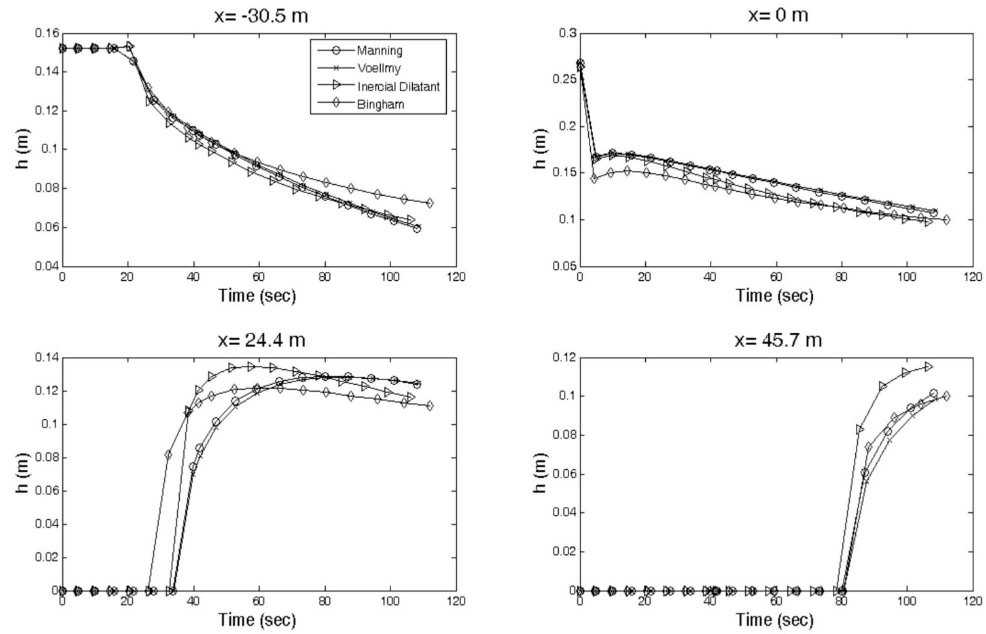

Figure 2. Evolution of computed depths at *x*=-30.5m, 0m, 24.4m and 45.7m for WES test 1.2.

FIELD SCALE CASE

The four rheological models, whose parameters were estimated from WES large scale laboratory experiments, are used to model two field scale cases: smooth and rough. The dam-break test case is defined as follow. A fluid front generated by an idealized dam failure propagates downstream in a 10m width channel, length 2L (L=500m), and slope θ. At t= 0s half of the channel is filled with fluid from x = -L to x = 0. This is shown in Figure 3.

The fluid is initially at rest. The depth of the layer of water downstream is h_{min} = 0.0001 m. The slope variation θ was in the interval [0.5º, 2º]. "Measurement stations" were placed at x= -0.5L, 0L, 0.3L and 0.6L. The results for the lowest (0.5º) and the highest (2º) slope considered in the smooth case are shown in Figures 4 and 5.

A good qualitative correspondence is observed for both cases (lowest and highest angles). Correlation indexes are shown in Tables 3 and 4.

The highest value for correlation index was of n=0.8646. This index is lower than the minimum value reached when rheological parameters for different models were obtained. Arrival times for fluid front are very similar in downstream stations. Maximum depth differences are in the ranges of 20%, and are obtained at the closest station to the channel exit.

Different results are found for the rough case. Depth hygrographs are shown in Figures 6 and 7.

In the small slope case, very different front velocities are found at arrival times to downstream measurement stations. Qualitatively, predictions for different models at the high slope case are no as good as the one for the smooth case. In particular, maximum depth differences are in ranges of 100% and very different flow depth are obtained along the path channel as a function of time. Correlation index for these cases are shown in Tables 5 and 6.

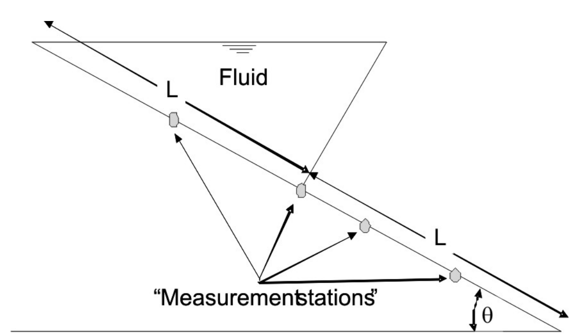

Figure 3. Diagram to define initial conditions of the problem of a dam failure.

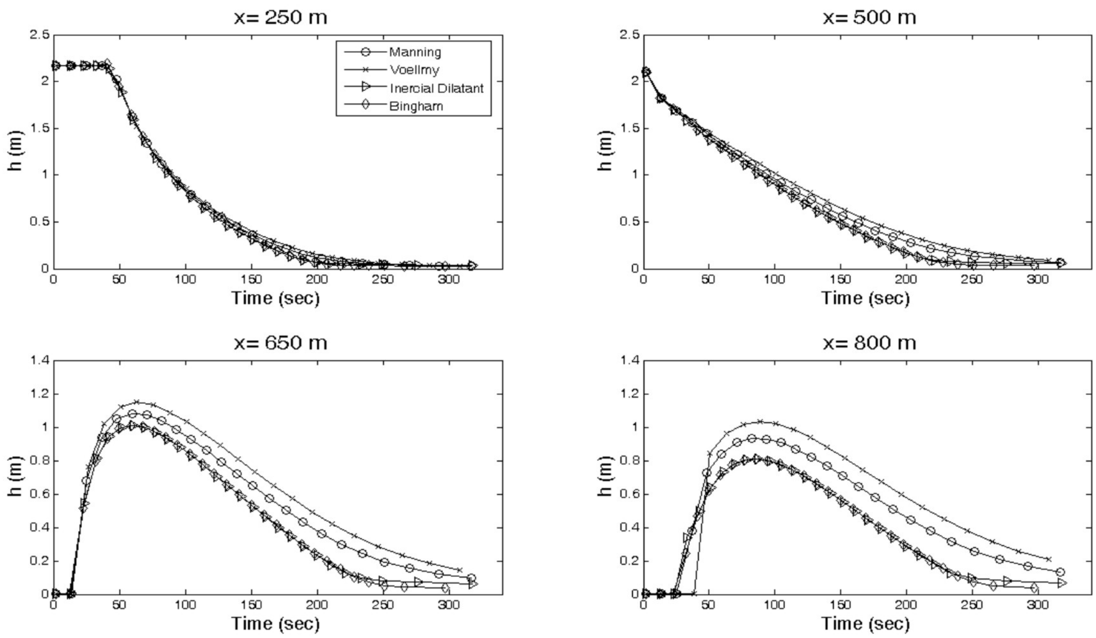

Figure 4. Evolution of depths at *x*=-250m, 0m, 150m and 300m for field scale: smooth/small slope case.

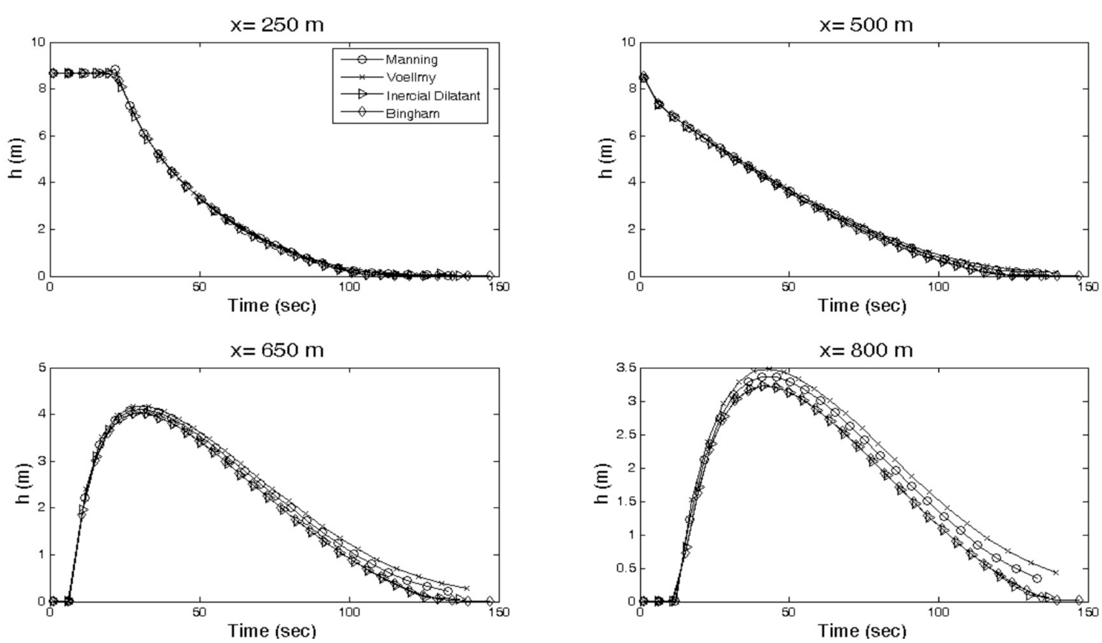

Figure 5. Evolution of depths at *x*=-250m, 0m, 150m and 300m for field scale: smooth/big slope case.

Table 3. Correlation index among depth predictions for rheological models in the field case with smooth surface and low slope (0.5°).

	Turbulent	Voellmy	Inercial	Bingham
	x = -250 m			
Manning	1	0.9997	0.9992	0.9991
Voellmy		1	0.9978	0.9977
Inercial			1	0.9999
Bingham				1
	x = 0 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9968	0.9934	0.9925
Voellmy		1	0.9819	0.9804
Inercial			1	0.9997
Bingham				1
	x = 150 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9882	0.9731	0.9698
Voellmy		1	0.9344	0.9302
Inercial			1	0.9991
Bingham				1
	x = 300 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9730	0.9410	0.9349
Voellmy		1	0.8720	0.8646
Inercial			1	0.9965
Bingham				1

Table 4. Correlation index among depth predictions for rheological models in the field case with smooth surface and big slope (2°).

	Turbulent	Voellmy	Inercial	Bingham
	x = -250 m			
Manning	1	0.9999	0.9999	0.9999
Voellmy		1	0.9997	0.9997
Inercial			1	1
Bingham				1
	x = 0 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9995	0.9992	0.9992
Voellmy		1	0.9976	0.9975
Inercial			1	1
Bingham				1
	x = 150 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9979	0.9965	0.9963
Voellmy		1	0.9896	0.9893
Inercial			1	0.9998
Bingham				1
	x = 300 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9951	0.9914	0.9912
Voellmy		1	0.9697	0.9761
Inercial			1	0.9997
Bingham				1

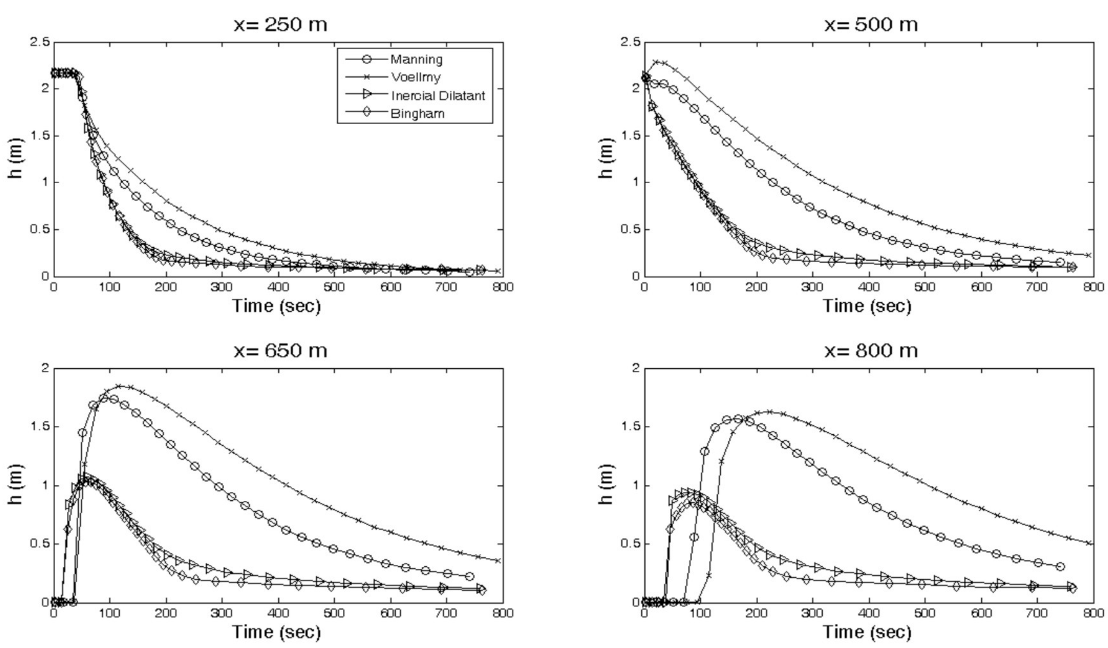

Figure 6. Evolution of depths at *x*=-250m, 0m, 150m and 300m for field scale: rough/small slope case.

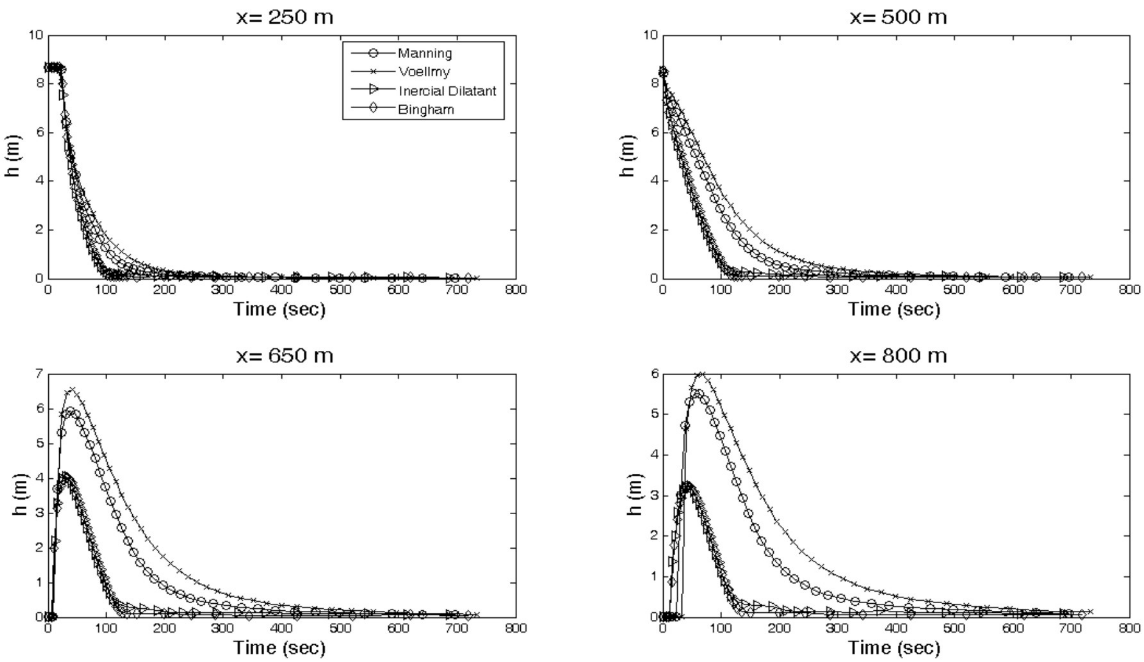

Figure 7. Evolution of depths at x=-250m, 0m, 150m and 300m for field scale (rough/big slope case).

Table 5. Correlation index between depth predictions
for rheological models in the field case with rough surface and small slope (0.5°).

	Turbulent	Voellmy	Inercial	Bingham
	x = -250 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9562	0.8972	0.8888
Voellmy		1	0.7716	0.7471
Inercial			1	0.9986
Bingham				1
	x = 0 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.8658	0.5965	0.5499
Voellmy		1	0.3918	0.3555
Inercial			1	0.9908
Bingham				1
	x = 150 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.7181	0.3324	0.2685
Voellmy		1	0.1140	0.0757
Inercial			1	0.9437
Bingham				1
	x = 300 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.5205	0.0862	0.0311
Voellmy		1	0.1300	0.1392
Inercial			1	0.8718
Bingham				1

Table 6. Correlation index among depth predictions for rheological models in the field case with rough surface and big slope (2°).

	x = -250 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9902	0.9740	0.9728
Voellmy		1	0.9393	0.9341
Inercial			1	0.9996
Bingham				1
	x = 0 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9653	0.8414	0.8319
Voellmy		1	0.7276	0.7148
Inercial			1	0.9984
Bingham				1
	x = 150 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.9334	0.6409	0.6259
Voellmy		1	0.4923	0.4756
Inercial			1	0.9945
Bingham				1
	x = 300 m			
	Turbulent	Voellmy	Inercial	Bingham
Manning	1	0.8912	0.4699	0.4498
Voellmy		1	0.2987	0.2780
Inercial			1	0.9874
Bingham				1

In the cases presented above no correlation can be established among some models. Figure 8 shows dependence among the worse correlation indexes in each measurement station as slope function.

Upstream depths are less influenced by the rheological model chosen. However, for the lowest angle, correlation index is lower than 0.82, which had been chosen as the reference value. Downstream, we found that the correlation indexes are too low for almost all cases considered.

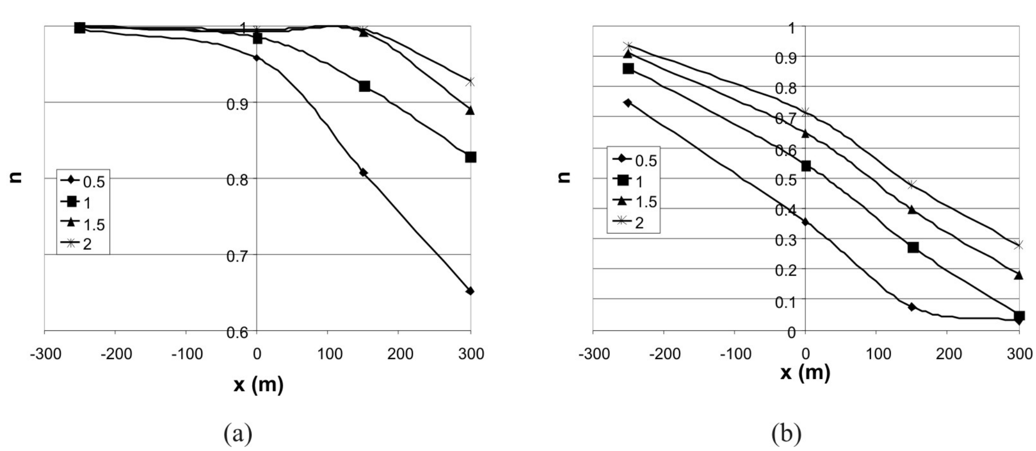

Figure 8. Evolution of correlation index for (a) smooth and (b) rough field cases as function of channel slope.

Thus, even though parameters for different rheological models can be adjusted for reproducing laboratory experiences at large scale, when they are employed for field scale test, predictions can be very different depending on particular characteristics of each case. It was shown that it is particularly true when channel roughness is appreciable.

In particular, for simulating debris-flow events, the approach to determine rheological parameters from laboratory experiences (although they used large facilities), does not seem to be adequate

CONCLUSIONS

In this paper a 1D numerical model was used for comparing predictions from four different resistance friction laws corresponding to four different rheological models. Parameters for each model were obtained by comparison with results from large scale laboratory experiments.

Even though rheological parameters could be adjusted in order to reproduce experimental data with good precision, when a field scale event is simulated, the predictions are very different among them. This indicates that calibration parameters may not be scale-independent.

Consequently, it is concluded that for simulating debris-flow events, the approach to determine rheological parameters from laboratory experiences, although it can be done in large facilities, does not seem to be extensible for field scale. Experiences in long flumes with mobile bed and adjustable slope should be done in order to simulate large front velocities and compare with predictions from different rheological models.

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