# FORMULATION OF A FINITE ELEMENT MODEL FOR THE STUDY OF VORTEX INDUCED VIBRATION IN SUBMARINE PIPELINES

*CARLOS BERRIO, EURO CASANOVA, ARMANDO BLANCO*
Departamento de Mecánica. Universidad Simón Bolívar.
e-mails: carjavier612@gmail.com, ecasanov@usb.ve, ajblanco@usb.ve

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## ABSTRACT

Vortex induced vibrations (VIV) is a condition that submarines pipelines could suffer over important periods of time and that may reduce in a relevant way the structure fatigue life, particularly, when they are not appropriately considered during the design stages. An approach to study the behavior of VIV in pipelines, of practical applicability for the industry, consists on the use of van der Pol oscillators adjusted to produce a "wake oscillator" model that simulates the fluid flow and can be coupled to a one-dimensional structural numerical model of the pipeline. In the specialized literature, many VIV studies based on the wake oscillator models can be found, but most of them use finite difference (FD) models of the pipeline. In this paper, the development of a numerical model for studying the dynamics of submarine pipelines subjected to VIV regime, using wake oscillators coupled with a finite element (FE) model of the pipeline and applying a Newmark scheme for the time integration, is presented. The proposed model was validated by comparing the response of a pipeline under different conditions and considering different effects (*e.g.* buoyancy, variations in fluid velocity, fluid-added mass, fluid damping, *etc.*) against the results obtained by FD based models presented in the literature. Results show that the proposed FE based model produces similar predictions for all the studied cases, with differences lower than 5%. In addition, the FE based model presents many advantages over FD based models, that are especially important for industrial applications e.g. simplicity in the introduction of different boundary and other effects, possibility of meshing of 3D pipeline configurations, etc.

*Keywords*: VIV, wake oscillator, submarine pipelines, van der Pol oscillator, finite element method

# FORMULACIÓN DE UN MODELO POR ELEMENTO FINITO PARA EL ESTUDIO DE VIBRACIONES INDUCIDAS POR VÓRTICES EN TUBERÍAS SUBMARINAS

### RESUMEN

Las vibraciones inducidas por vórtices (VIV) son una condición a la que pueden estar sometidas las tuberías submarinas durante importantes periodos de tiempo y que puede reducir de forma significativa la vida a fatiga de la estructura, particularmente cuando dicha condición no se ha tomado en cuenta en las etapas de diseño. Un enfoque utilizado para estudiar el comportamiento de tuberías sometidas a VIV, de aplicación a casos de interés industrial, consiste en el uso de múltiples osciladores de van der Pol, ajustados para producir modelos de "oscilador de estela" que simulan el flujo del fluido y que pueden acoplarse a un modelo numérico unidimensional de la estructura de la tubería. En la literatura especializada se pueden encontrar muchos estudios VIV basados en el modelo de oscilador de estela, sin embargo, la mayoría de ellos utilizan modelos estructurales de la tubería en diferencias finitas (DF). En este trabajo se desarrolla un modelo numérico para el estudio de la dinámica de tuberías submarinas sometidas al régimen VIV, utilizando osciladores de estela acoplados a un modelo de elementos finitos (EF) de la estructura y aplicando un esquema de Newmark para la integración temporal. El modelo propuesto fue validado comparando la respuesta de una tubería en diferentes condiciones y considerando distintos efectos (e.g. flotabilidad, variaciones en la velocidad del fluido, masa agregada por fluido, amortiguación del fluido, etc.) respecto a los resultados obtenidos por algunos modelos FD descritos en la literatura. Los resultados muestran que el modelo de EF propuesto produce predicciones similares para todos los casos estudiados, con diferencias relativas inferiores al 5 %.

*Palabras clave*: VIV, oscilador de estela, tuberías submarinas, oscilador de van der Pol, método del elemento finito

## INTRODUCTION

With a world population over seven billion and growing, and limited access to natural resources, engineers worldwide are focused in obtaining the maximum advantage of these resources through the increase of efficiency in all processes involved with energy production, conversion, and transportation. In the petroleum industry, there are several options to transport oil from facilities in land or offshore oilfields to processing plants. Submarine pipelines are one of these options and one whose usage is increasing, because it has some advantages *e.g.* low operation and maintenance costs, once installed. However, submarine pipelines are subject to some aggressive factors related to the marine environment. One of these factors are the sea currents that act directly over the pipelines producing a dynamic response of the structure known as vortex induced vibrations (VIV).

VIV is a condition that submarines pipelines could suffer over important periods of time and may reduce in a relevant way the structure fatigue life, particularly, when they are not effectively considered during the design stages. The proper way to estimate external fluid effects around submarines pipelines is through the numerical solution of Navier–Stokes equations for the fluid flow coupled with the government equation that describes the dynamics of the structure, *i.e.* fluid-structure interaction. However, this approach is not of practical applicability for the industry because the numerical modeling of fluid-structure interaction is very complex and requires huge computational resources and time (Lucor *et al.*, 2001). An alternative way to study the behavior of VIV in pipelines consists on the use of van der Pol oscillators adjusted to produce a "wake oscillator" model that simulates the fluid flow and can be coupled to a one-dimensional structural numerical model of the pipeline. This approach has been validated with experimental and numerical results using computational fluid dynamics (CFD) and it offers a relevant decrease in computational cost, contrasted with such CFD analysis, with a fairly good precision in the predictions (Facchinetti *et al.*, 2001; Chaplin *et al.*, 2005 and Gabbai & Benaroya, 2005).

In the specialized literature, many VIV studies based on the wake oscillator model can be found, but most of them use finite difference (FD) models to discretize the motion equation of the pipeline, consequently they are limited to study straight pipe sections and VIV effects in only one direction (Balasubramanian, 1997; Sarpkaya, 2004 and Williamson, 2004). However, FD models may not be the best choice to solve more complex problem such as three dimensional (3D) studies or curved pipelines systems. On the contrary, including 3D features using a finite element (FE) model is relatively simple and additionally the handling of different boundary conditions, loads, *etc.*, is greatly simplified. To the authors' knowledge, most of the few FE models presented in the literature don't model the fluid but rather introduce the fluid effect in the pipeline through a force term (*e.g.* Huera *et al.*, 2006; and Ulveseter *et al.*, 2017). Those who model the fluid flow by means of wake oscillator models don't show how the discretization of this aspect is taken into account (*e.g.* Bai and Qin, 2014).

Considering the wide range of piping systems subject to VIV that could be defined and studied, a wake oscillators coupled to a FE model was developed to simulate the dynamics of submarine pipelines subjected to VIV regime. In order to validate the described model, as well as its utility and limitations, the response of a pipeline under different conditions and considering different effects, was calculated and the results were compared against those obtained by FD based model presented in literature.

## MATHEMATICAL MODEL

This section describes the mathematical model developed to analyze the VIV response in submarine pipelines.

### Fluid model

In VIV analysis, the main purpose of modeling the fluid flow is to simulate the forces generated by the flow acting on the structure and how the flow is in return influenced by structure movements. From the structural point of view, only the flow forces are of interest, therefore no detailed analysis of the flow is required, and the use of semiempirical models, based on simple parameters (force coefficients), describing in a simple way the flow, is possible. According to Chaplin *et al.* (2005), a semiempirical model could be as successful as a CFD simulation in VIV response of vertical beam which staggered currents.

One of the most common semiempiric approaches is the van der Pol oscillator, which was proposed for the first time by Birkoff and Zarantanello (1957) to model the excitations that can suffer a submerged cylinder due a flow. Those authors presented an oscillator model that quantitatively, and in some cases, qualitatively, reproduce some of VIV aspects observed in experiments for rigid cylinders elastically supported.

The van der Pol oscillator is a non-conservative second order system with a non-linear term proportional to velocity. The non-linear term either dissipates energy, for high oscillation amplitudes, or generates energy, for low oscillation amplitudes. As a result, oscillations are produced around a state or cycle, in which the energy generated and dissipated tend to be balanced. The simplest van der Pol oscillator depends only on one parameter affecting the non-linear term (ε), and is described by van der Pol (1920):

$$\ddot{x}-\varepsilon\left(1-x^2\right)\dot{x}+x=0 \tag{1}$$

where $x$ is a particular function (*e.g.* displacement) and dot over $x$ represents time derivatives.

Wake oscillator, refers to a family of models based on the properties of the van der Pol equation, and adjusted to model the fundamental phenomenon of VIV (Gabbai and Benaroya, 2005). Due its simplicity, wake oscillator models may be easily extended to analyze VIV in 2D and 3D problems. This kind of models are useful when computational resources demands are important, as in the case of fluid-structure interaction simulation, especially on 3D models (Facchinetti *et al.*, 2004).

Mathelin and Langre (2005) used a van der Pol forced model similar to the one shown in Eq. (2) to describe submerged cylinder dynamics:

$$\frac{\partial^2 q}{\partial t^2}+\Omega\epsilon\left(q^2-1\right)\frac{\partial q}{\partial t}+\Omega^2 q=\frac{A}{D}\frac{\partial^2 y}{\partial t^2} \tag{2}$$

where $t$ represents the time, $q$ is a dimensionless variable corresponding to a fluctuating lift coefficient in the structure, $A$ is the force parameter, Ω is the Strouhal pulsation of vortex shedding, $y$ is the beam deflection and $D$ is the external pipe diameter.

**Structural model**

In order to model the pipeline dynamic behavior when subjected to flow excitations, the pipeline was considered as a long-thin hollow cylinder with an incident flow, as showed in Figure 1.

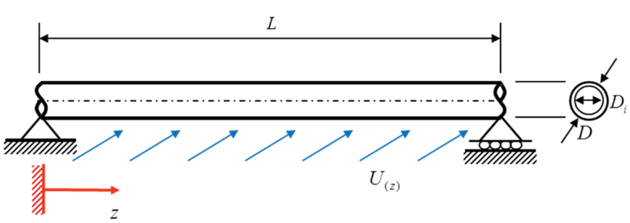


***Figure 1.*** *Physical model considered*

The pipeline, subjected to an incident flow with velocity $U$, is assumed to only oscillate in the cross-flow direction as shown in Figure 2. Pipeline oscillations on the in-line direction were not considered in this work; however, they can be easily incorporated in the final model.

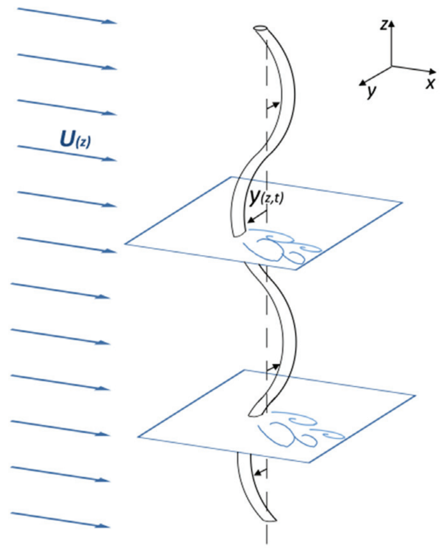


***Figure 2.*** *Pipeline vibrating in the cross-flow direction*

The pipeline movement can be modeled by the Bernoulli's beam equation including the tension term and considering that material and structural properties are constant over the length:

$$m\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}-T_o\frac{\partial^2 y}{\partial z^2}+EI\frac{\partial^4 y}{\partial z^4}=f \tag{3}$$

where $y$ represents the deflection of pipe for each point of coordinate $z$; $m$, $c$, and $f$ are the mass, damping and force terms, respectively; $T_o$ is the tension per unit of length applied to the beam and $EI$ is the stiffness per unit of length of the beam. In VIV analysis, $m$, $c$ and $f$ terms have particular expressions.

***Mass term***. This term includes the mass per unit of length of the structure $m_s$ (*i.e.* pipe and its content) and the fluid "added mass" $m_f$ which models fluid inertial effects. These inertial effects are expressed as:

$$m_f=\frac{1}{4}\pi C_m \rho D^2 \tag{4}$$

being $C_m$ = 1, the added mass coefficient for a circular cylinder (Blevins, 1990) and ρ the fluid density.

Finally:

$$m = (m_s + m_f) = \left(m_s + \frac{1}{4}\pi C_m \rho D^2\right) \tag{5}$$

***Damping term***. In addition to any structural damping contribution (neglected in this work), there is damping due to the movement of the pipeline in the external flow (Violette *et al.* 2007) which is related to the drag:

$$c = \frac{1}{2} C_d U \rho D \tag{6}$$

where $C_d$ is the drag coefficient and $U$ the flow velocity.

***Force term.*** Forces produced by the fluid are related to lift and buoyancy effects (Bossio *et al.* 2014):

$$f = \frac{1}{4} C_{Lo} D U^2 \rho q - g(m_s + m_f)\left(1 - \frac{\rho A_s}{m_s}\right) \tag{7}$$

with $C_{Lo} = 0.3$, which represent the amplitude of fluctuating lift for a fixed cylinder, $g$ the gravity value and $A_s$ the transversal section of the pipe.

**Coupled model**

Coupled model results from the simultaneous resolutions of equations (2) and (3), including all the terms previously defined:

$$\begin{cases} m\dfrac{\partial^2 y}{\partial t^2} + c\dfrac{\partial y}{\partial t} - T_o\dfrac{\partial^2 y}{\partial z^2} + EI\dfrac{\partial^4 y}{\partial z^4} = f \\ \dfrac{\partial^2 q}{\partial t^2} + \Omega\epsilon(q^2 - 1)\dfrac{\partial q}{\partial t} + \Omega^2 q = \dfrac{A}{D}\dfrac{\partial^2 y}{\partial t^2} \end{cases} \tag{8}$$

Pipeline structure was discretized by means of FE method using beam elements based on Euler-Bernoulli beam theory, with two nodes and three degrees of freedom (DOFs) per node, corresponding to the beam axial displacement, the beam deflection and beam rotation, as referenced by multiples books *e.g.* Chandrupatla & Belegundu (2002). The fluid, modeled by the van der Pol oscillators in equation (8), was considered to act along the pipeline, coupled with the pipeline through the beam nodes, specifically through the beam deflection DOFs. Therefore, the dynamics of the system, considering *n* DOFs for the structure and *p* DOFs for the fluid, is described by:

$$\begin{cases} M_{nn}\ddot{x}_{n(t)} + C_{nn}\dot{x}_{n(t)} + K_{nn}x_{n(t)} = B_{np}q_{p(t)} - w_n \\ \ddot{q}_p + D_{pp}[Diag(q_p q_p^T) - I_p]\dot{q}_p + G_{pp}q_p = A_{pn}\ddot{x}_n \end{cases} \tag{9}$$

where $p$ and $n$ sub-indexes represent the corresponding matrix and vector dimensions; $x$, $\dot{x}$, $\ddot{x}$ are the vectors of nodal displacements, velocities and accelerations for the pipeline, respectively; $q$, $\dot{q}$, $\ddot{q}$ are the vector of nodal values, and temporal rates of the non-dimensional variable related to lift respectively, **I** is the identity matrix; and Diag(·) is a function that generates a diagonal matrix formed only by the diagonal elements of the matrix used as argument.

The mass, damping and stiffness matrixes are defined in equations (10), (12) and (14), respectively.

$$M = M^s + M^f \tag{10}$$

with $\mathbf{M}^S$ the assembled FE mass matrix of the pipeline, and:

$$M^f = M^f_{i,j} = \begin{cases} 0 & for\ j \neq i \\ \dfrac{1}{4}\pi C_m \rho D^2 & for\ j = i \end{cases} \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{matrix} \tag{11}$$

$$C = C^s + C^f \tag{12}$$

where $\mathbf{C}^S = \mathbf{0}$ is the structural assembled damping matrix (not considered in this work), and:

$$C^f = C_{j,i} = \begin{cases} 0 & for\ j \neq i \\ \dfrac{1}{2} C_d U \rho D & for\ j = i \end{cases} \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{matrix} \tag{13}$$

$$K = K^s_{flex} + K^s_{T_o} \tag{14}$$

where $\mathbf{K}^S_{flex}$ and $\mathbf{K}^S_{To}$ are the pipeline assembled stiffness matrixes due to flexion and tension, respectively.

The matrixes **B**, **A**, **G** and **D** are defined as:

$$K = K^s_{flex} + K^s_{T_o} \tag{14}$$

$$B = B_{j,i} = \begin{cases} 0 & for\ j \neq i \\ \dfrac{1}{4} C_{Lo} D U^2 \rho & for\ j = i \end{cases} \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{matrix} \tag{15}$$

$$A = A_{j,i} = \begin{cases} 0 & for\ j \neq i \\ \dfrac{A_i}{D_j} & for\ j = i \end{cases} \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{matrix} \tag{16}$$

$$D = Diag(\Omega_i \epsilon_i) \tag{17}$$

$$G = Diag(\Omega_i^2) \tag{18}$$

Finally, buoyancy effects are represented by vector **w** as:

$$w = g\,Diag\left(\left(M^s + M^f\right)\left(I_n - \rho\,A_s\,M^{s-1}\right)\right) \tag{19}$$

## VALIDATION OF THE PROPOSED MODEL

This section presents the validation of the FE model proposed in this paper. For all the cases shown hereafter, spatial convergence was verified by comparison of the first 20 normal frequencies against analytical values. Due to the fact that an experimental validation in this subject is complex and very expensive, a numerical validation, by comparison against results from the literature, was performed. For all the validations, the coupled model (equation 9) was integrated in a direct way using the Newmark method (Newmark, 1959), due to its simplicity, stability and broad use in the resolution of structural dynamic problems.

### Response of a horizontal pipeline subjected to different VIV parameters

In order to validate that the FE model proposed in this paper (denoted by Mfem) captures appropriately some of the effects that may be present in the VIV phenomenon (*i.e.* gravity, fluid velocity and fluid added mass), six cases, not necessarily representing real physical conditions, were analyzed. These cases are described in Table 1. The FD model used by Bossio *et al.* (2014), denoted by Mdf, was employed to compare predictions for each of these cases.

**Table 1.** Description of cases for comparison

| Case | Gravity, $g\left[\frac{m}{s^2}\right]$ | Flow velocity, $U\left[\frac{m}{s}\right]$ | Added mass, $m_f\left[\frac{kg}{m}\right]$ |
|---|---|---|---|
| 1 | ○ | ○ | ○ |
| 2 | ● | ○ | ○ |
| 3 | ● | ● | ○ |
| 4 | ● | ● | ● |
| 5 | ○ | ● | ● |
| 6 | ○ | ● | ○ |

(●) Parameter considered, (○) Parameter not considered.

**Table 2**. Pipeline physical parameters

| $D$ [m] | $L$ [m] | $D$ [m] | $E$ [GPa] | $\rho_s$ $\left[\frac{kg}{m^3}\right]$ | $T_o$ [N] |
|---|---|---|---|---|---|
| 0.508 | 100 | 0.482 | 200 | 7850 | 0 |

**Table 3**. Fluid flow physical parameters

| $\rho_f$ $\left[\frac{kg}{m^3}\right]$ | $U$ $\left[\frac{m}{s}\right]$ | $C_m$ | $S_t$ | $C_{Lo}$ | $C_d$ | $\epsilon$ | $A$ |
|---|---|---|---|---|---|---|---|
| 1000 | 2 | 1 | 0.2 | 0.3 | 2 | 0.3 | 12 |

Note: $S_t = \frac{\Omega D}{U}$

In all six cases, initial conditions were set to $10^{-3}$ m of displacement and null velocity, for the structure respectively, and null values for the flow variable and its rate respectively. On Tables 2 and 3, the physical parameters defining the structure and the fluid are shown, respectively. The time step employed in all cases was $10^{-4}$ s, in order to minimize possible errors, due to the nonlinear nature of the problem. Figures 3 and 4 show the pipeline midpoint displacement in time for Case 4 and Case 5 conditions, respectively. It is worth noting that these two cases differ only in the presence of gravity. In these figures, it can be observed that the predictions of both models, Mdf and Mfem, are practically identical, although a slight shift in time and amplitude is observed in Figure 3b and 4b.

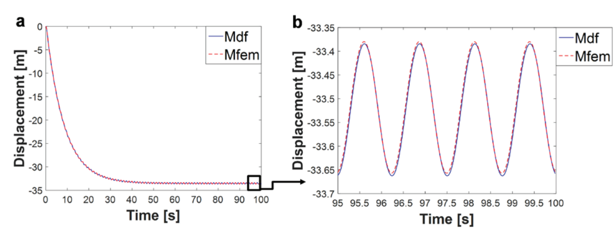


***Figure 3.*** *Pipeline midpoint displacement for Case 4. (a) Total time; (b) Detailed view of the last 5 s*

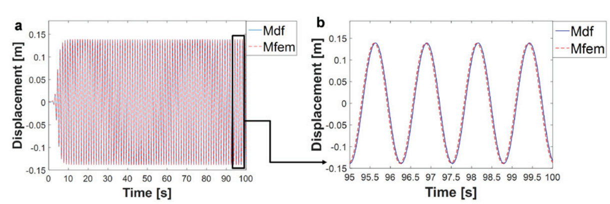


***Figure 4.*** *Pipeline midpoint displacement for Case 5. (a) Total time; (b) Detailed view of the last 5 s*

In addition to these figures, Table 4 and Table 5 show the root mean square (RMS) values and the spectrum maximum amplitude for the pipeline midpoint displacements, for the six Cases presented in Table 1, respectively. These tables show that for all the Cases, differences between Mdf and Mfem predictions are lower that 5 %.

**Table 4**. RMS of pipeline midpoint displacement

| Case | Mdf [m] | Mfem [m] | Relative error [%] |
|---|---|---|---|
| 1 | $8.90x10^{-4}$ | $8.52x10^{-4}$ | 4.22 |
| 2 | 17.78 | 17.57 | 1.20 |
| 3 | 13.56 | 13.56 | 0.015 |
| 4 | 31.30 | 31.30 | 0.014 |
| 5 | 0.095 | 0.096 | 0.63 |
| 6 | 0.023 | 0.023 | 3.43 |

**Table 5**. Maximum amplitude of the pipeline midpoint displacement spectrum

| Case | Mdf [m] | Mfem [m] | Relative error [%] |
|---|---|---|---|
| 1 | 1.22x10-3 | 1.24x10-3 | 1.39 |
| 2 | 13.98 | 14.17 | 1.36 |
| 3 | 26.56 | 26.55 | 0.038 |
| 4 | 61.26 | 61.25 | 0.016 |
| 5 | 0.14 | 0.13 | 1,55 |
| 6 | 0.023 | 0.022 | 4.34 |

**Response of a tensioned cable subject to a uniform flow**

The finite element model proposed in this paper was also validated against some of the results presented by Violette *et al.* (2007). One of the cases they studied was an infinite tensioned cable, with no flexural stiffness and without structural damping, subjected to a uniform flow and imposing spatial periodic conditions as the cable extreme boundaries conditions. The physical parameters that define this case are shown in Table 6, in terms of the non-dimensional magnitudes used by Violette *et al.* (2007).

**Table 6**. Non-dimensional parameters used by Violette *et al.* (2007)

| Parameter | Value |
|---|---|
| $S_t=\frac{\Omega D}{U}$ | 0.16 |
| $\frac{L}{D}$ | 8π |
| $\frac{(m_s+m_f)}{\rho D^2}$ | 1.785 |
| $\frac{T_o}{(m_s+m_f)\Omega^2 D^4}$ | 4 |
| $\frac{EI}{(m_s+m_f)(\Omega D)^2}$ | 0 |

According to Violette *et al.* (2007), a random noise with amplitude of order $10^{-3}$ applied to the fluid variable $q$ was used as initial condition. Initial displacements and velocities of the structure were considered null. For the spatial discretization, a model with 252 nodes was considered, and for the temporal discretization, a dimensionless total time of 600 and time step of 0.01 were employed. In addition, the Reynolds number was set to 100.

Figure 5 shows the responses for the infinite cable modeled with a direct numerical simulation (DNS) used by Newman and Karniadakis (1997), wake oscillator with FD, used by Violette *et al.* (2007) and the FE model (Mfem) developed in this paper. Violette *et al.* (2007) show in their article Figures 5a and 5b, both of them, presenting an initial movement along the cable in the form of a standing wave that, in the vicinity of 10 cycles, turns into a traveling wave. As can be seen in Figure 5c, both features are well reproduced by the model proposed in this paper (Mfem).

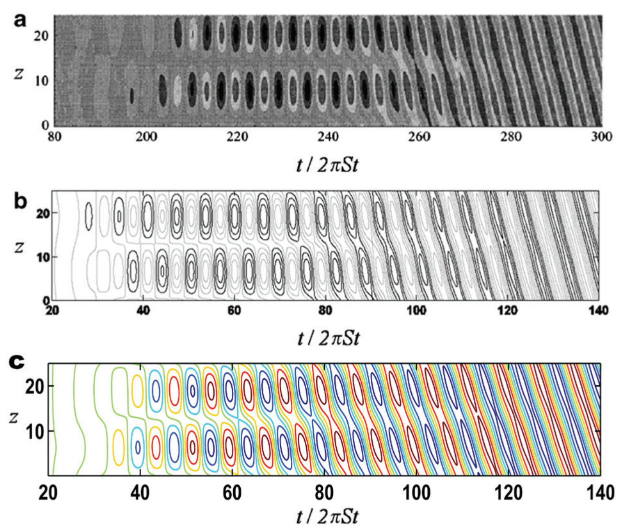


***Figure 5.*** *Contour displacement along the cable for infinite tensioned cable under uniform flow (a) DNS (Newman, 1997); (b) Wake oscillator (Violette et al., 2007); (c) Mfem*

**Response of a beam subjected to a non-uniform flow**

Another case studied by Violette *et al.* (2007) is about a vertical tensioned beam pinned at both ends and subjected to a linear flow profile that simulates the velocity profile that is normally considered in a riser study. For this case, Violette *et al.* (2007) compare their Wake oscillator model against DNS model results from Lucor *et al.* (2006). The tensioned beam has zero structural damping and 2000 nodes were used for the spatial discretization. Dimensionless time and time step of 1600 and 0.001 were considered, respectively. As initial condition, a random noise of order $10^{-3}$ amplitude on the fluid variable $q$ and zero displacement and velocity for the structure, was set. Additional dimensionless parameters defined by Violette *et al.* (2007) are shown in Table 7. Regarding the flow profile, a linear fluid flow with a 70% variation from the maximum velocity value (*i.e.* $\Delta U = 0.7U_{max}$) and a Reynolds number of 1000 used to define $U_{max}$, were considered.

**Table 7**. Non-dimensional parameters used by Violette *et al.* (2007)

| Parameter | Value |
|---|---|
| $S_t = \frac{\Omega D}{U}$ | 0.2 |
| $\frac{L}{D}$ | 2028 |
| $\frac{(m_s + m_f)}{\rho D^2}$ | 2.785 |
| $\frac{T_o}{(m_s + m_f)\Omega^2 D^4}$ | 23.6 |
| $\frac{EI}{(m_s + m_f)(\Omega D)^2}$ | 303 |

Figure 6 shows the RMS values of the displacement obtained by the three models (*i.e.* Lucor *et al.* (2006), Violette *et al.* (2007), and Mfem). From the responses observed in Figure 6a and Figure 6b, Violette *et al.* (2007) describe "good similarities" between DNS and Wake oscillator responses, focusing in the behavior of the standing waves near the beam's end and how the Wake oscillator model predicts the traveling waves away from the beam ends. Figure 6c shows the Mfem prediction and it could be appreciated a similar behavior on the response near and far away from the beam's end. Note however, that some differences are observed between the predictions of the three models compared, which are probably to the fact that none of models has reached a steady state.

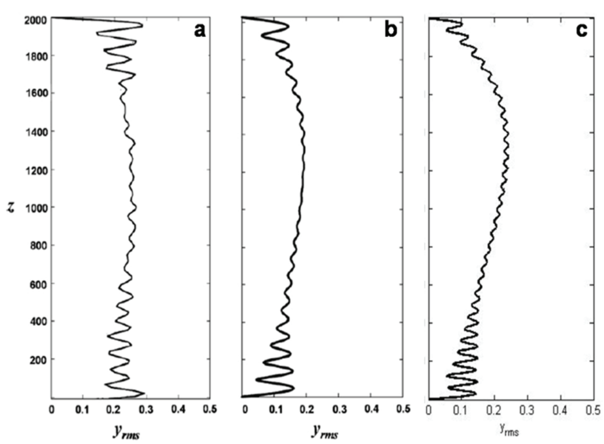


***Figure 6.*** *RMS value of the displacement along a finite tensioned beam under linearly sheared flow (a) DNS (Lucor et al., 2006); (b) Wake oscillator (Violette et al., 2007); (c) Mfem*

**Response of a pipeline in the lock-in region**

The lock-in or synchronization phenomenon, widely analyzed in VIV studies, consists in the self-excitement of the structure by the synchronization of vortex shedding frequency and natural frequency of the structure. The resulting vibration in this region has a frequency close to the natural frequency of the pipeline. According to de Langre (2006) this phenomenon conveys an enormous amount of energy to the structure, thus producing high amplitudes. On the other hand, as a result of the synchronization of the vortex shedding frequency and natural frequency of the structure, a highly non-linear (jump-phenomenon) and path-dependent behavior is observed in this region.

For lock-in studies a reduced velocity, which relates the flow velocity with the natural frequency of the pipe, is defined as:

$$U_r = \frac{2\pi}{\Omega_s}\frac{U}{D} \quad (20)$$

where $\Omega_S$ is the first natural frequency of the pipe and the flow velocity $U$ is defined as:

$$U = \frac{\Omega_f D}{2\pi S_t} \quad (21)$$

Upon replacing equation (21) in equation (20), the reduced velocity is:

$$U_r = \frac{\Omega_f}{\Omega_s}\frac{1}{S_t} \approx \frac{1}{S_t} \quad (22)$$

In order to study the capabilities of the proposed model in the lock-in region, a horizontal pipeline defined by parameters of Tables 2 and 3, was studied. According to Bokaian (1994) the structure responses under lock-in regime, once they have reached the maximum amplitude, start to fall in the range [$6.5 < U_r < 8$], therefore to obtain the response of the pipeline under lock-in condition, the reduced velocity ($U_r$) was varied in a defined range [0 - 19.37] and, since the behavior is path-dependent, this variation was done in ascending and descending order of relative velocity variations.

Figure 7 shows the steady state maximum midpoint amplitudes of the pipeline using the FD model described by Bossio *et al.* (2014) and the FE model (Mfem) proposed in this work. The similarities between both models are easily appreciated, furthermore, responses show the behavior described by Bokaian (1994) and they begin to decline after reach the maximum amplitude of 0.142 m at $U_r = 5.8$.

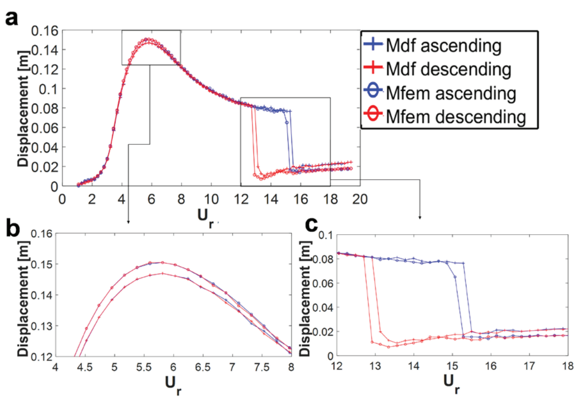


***Figure 7.*** *Lock-in regime using Mdf and Mfem (a) Lock-in in the middle of the pipe; (b) detail of the maximum amplitude; (c) detail of the jump*

In Figure 7a some differences between the responses of both models are observed. Figure 7b, shows the differences in the maximum amplitudes, and Figure 7c shows the differences in the range where the jump-phenomenon occurs, nevertheless, the relative differences in both cases are no greater than 3 %.

## CONCLUSIONS

This paper presented a methodology to obtain a FE model (Mfem) to predict the VIV that may experience submarine pipelines when submitted to the action of sea currents. The proposed model was validated against a FD model used by Bossio *et al.* (2014) and against results presented by Violette *et al.* (2007), Newman and Karniadakis (1997) and Lucor *et al.* (2006), showing similar responses from a qualitative and quantitative point of view and obtaining differences no greater than 5% in all studied cases.

For all the validation cases studied, the FE model proposed herein prove to be an efficient and flexible tool to predict the responses. Furthermore, note that the DNS results presented in some of the validations, which require huge amounts of computational resources, are still of no practical interest for the industry due to their computation and time cost.

The model proposed herein opens the possibility to simulate complex field cases were 3D pipeline configurations are common and other effects, besides VIV, should be taken into account *e.g.* dynamic excitations due to internal flow, soil interaction, buckling, *etc.*

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