SPE-169300-MS

A Simplified Computer Model for Determination of Cuttings Concentration and Wellbore Cleaning

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This paper was prepared for presentation at the SPE Latin American and Caribbean Petroleum Engineering Conference held in Maracaibo, Venezuela, 21–23 May 2014.

Abstract

Plugging of cuttings in the wellbore is a common problem during drilling processes. Some authors have analyzed settling or slip velocity of particles in a liquid both analytically and experimentally. The concentration of cuttings throughout the well has been also analyzed. It has been proven that numerical models have the potential to help predict the performance of the cuttings concentration for solving this problem.

A computer model for determining the cuttings concentration for the different processes applied during wellbore drilling is presented. It is a useful tool for the drilling engineer to predict and analyze the drilled cuttings transport to assure efficient wellbore cleaning during operations. The model is based on central difference, and non-Newtonian power low fluids can be used. The effect of the cuttings interaction for different concentration in the settling velocity was determined experimentally. The model was designed to allow the user to calculate the cuttings bed formation, determine whether the bed remains stationary or build ups and analyze the position of the cuttings bed formed and the height of the cuttings bed layer.

Using a typical case study, the wellbore cuttings concentration and the height of the cutting bed are shown for the different processes; i.e. during hole-cleaning and shut down. The effect of operating parameters, wellbore geometry, non-Newtonian fluid parameters and cutting characteristics was analyzed. This computer model is a helpful tool to predict stuck problems and speed up wellbore cleaning processes.

Particles Transport Model

Conservation of mass

Conservation of mass is the basic transport equation for the analysis of the concentration of particles in a liquid flow. The total material flowing into an element of the system (see Fig 1) less the total output flow equals the accumulation of material. If the particles are assumed incompressible, this statement can be written as follows (Civan):

$$\overline{A}\Delta x\frac{C^{t+\Delta t}-C^{t}}{\Delta t}=-(AVC)_{x+\Delta x}+(AVC)_{x}-\frac{F}{\Delta t} \quad \text{.....(1)}$$

where the left hand side is the rate of accumulation, and the right hand side accounts for incoming particles through the surface located at the element x, and outcoming particles flowing through: the surface located at $x+\Delta x$, and the formation.

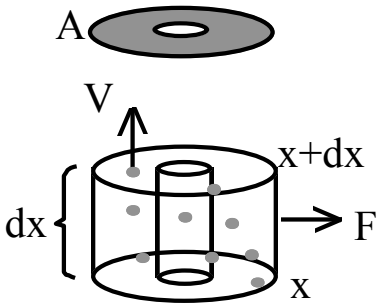

Fig. 1-Sketch of a control volume (basic element)

For the cuttings analysis, where the particles are usually larger than the pore size, F can be assumed negligible. The cross-sectional area in each element can be assumed constant. Then, equation (1) can be written:

$$\frac{C^{t+\Delta t}-C^{t}}{\Delta t}=-\frac{(VC)_{x+\Delta x}-(VC)_{x}}{\Delta x} \quad \text{.....(2)}$$

Let's define $\frac{\Delta C_j}{\Delta t}=\frac{C^{t+\Delta t}{}_j-C^{t}{}_j}{\Delta t}$ and $\frac{\Delta(VC)_x}{\Delta x}=\frac{(VC)_{x+\Delta x}-(VC)_x}{\Delta x}$, and take the limit when

$\Delta x \& \Delta t \to 0$. The equation can be written in the differential form:

$$\frac{\partial C}{\partial t}=-\frac{\partial CV}{\partial x} \quad \text{.....(3)}$$

The right hand side can be written: $\frac{\partial CV}{\partial x}=\frac{C\partial V}{\partial x}+\frac{V\partial C}{\partial x}$

As shown later, the relation between the velocity and the concentration can be calculated. It is convenient to apply the chain rule $\frac{\partial V}{\partial x}=\frac{dV}{dC}\frac{\partial C}{\partial x}$, which leads to:

$$\frac{\partial C}{\partial t}=-(C\frac{dV}{dC}\frac{\partial C}{\partial x}+\frac{V\partial C}{\partial x}) \quad \text{or} \quad \frac{\partial C}{\partial t}=-(V+C\frac{\partial V}{\partial C})\frac{\partial C}{\partial x}$$

Let's define $\alpha=V+C\frac{\partial V}{\partial C}=\alpha(C)$ .....(4)

which is a parameter with velocity dimension and is a function of the concentration. Then, the equation becomes:

$$\therefore \frac{\partial C}{\partial t}=-\alpha\frac{\partial C}{\partial x} \quad \text{.....(5)}$$

Note that when α is a constant, that is no slip or very low concentration, eq. (5) becomes $\frac{\partial C}{\partial t}=-V\frac{\partial C}{\partial x}$. Any function of the concentration with respect to time at the bottom of the well (BC's) will travel along the well with the same shape at a velocity V .

Finite Difference Solution

A diagram of the discretization is shown in Fig. 2. The method chosen for the discretization of the time component was Stone & Brian, centered in time and weighted in space. It is defined as:

$$\frac{\partial C_j}{\partial t}=w_1\frac{(C^{n+1}{}_{j-1}-C^{n}{}_{j-1})}{\Delta t}+w_2\frac{(C^{n+1}{}_{j}-C^{n}{}_{j})}{\Delta t}+w_3\frac{(C^{n+1}{}_{j+1}-C^{n}{}_{j+1})}{\Delta t} \quad \text{.....(6)}$$

$$w_1+w_2+w_3=1 \quad \text{.....(7)}$$

Where w_1, w_2, w_3 are weighting factors determined for best numerical performance.

Note that defining $\theta=w_1+w_3$, leads to $w_2=1-\theta$. If $w_1=w_3$, the eq. (6) becomes:

$$\frac{\partial C_j}{\partial t} = (1-\theta)\frac{(C^{n+1}{}_j - C^n{}_j)}{\Delta t} + \theta\left[\frac{(C^{n+1}{}_{j-1} - C^n{}_{j-1})}{2\Delta t} + \frac{(C^{n+1}{}_{j+1} - C^n{}_{j+1})}{2\Delta t}\right] \quad (6.1)$$

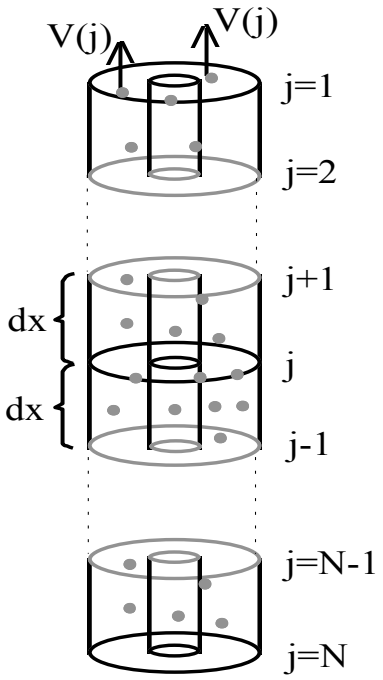

Fig. 2-Diagram of the discretization of the well.

The equation chosen for the discretization of the distance component is also centered in time and weighted in space:

$$-\frac{\partial C_j}{\partial x} = \frac{f_1}{2}\left[\frac{(C^n{}_{j+1} - C^n{}_j)}{\Delta x} + \frac{(C^{n+1}{}_{j+1} - C^{n+1}{}_j)}{\Delta x}\right] + \frac{f_2}{2}\left[\frac{(C^n{}_j - C^n{}_{j-1})}{\Delta x} + \frac{(C^{n+1}{}_j - C^{n+1}{}_{j-1})}{\Delta x}\right] \quad (8)$$

$$f_1 + f_2 = 1 \quad (9)$$

f_1, f_2 are also weighting factors determined for best performance.

Note that when $f_1 = f_2 = 1/2$, eq. (8) becomes

$$-\frac{\partial C_j}{\partial x} = \frac{(C^n{}_{j+1} + C^{n+1}{}_{j+1})}{4\Delta x} + \frac{(C^n{}_{j-1} + C^{n+1}{}_{j-1})}{4\Delta x} \quad (8.1)$$

Equation (8) can be written:

$$-2\Delta x\frac{\partial C_j}{\partial x} = f_1\left[C^n{}_{j+1} - C^n{}_j + C^{n+1}{}_{j+1} - C^{n+1}{}_j\right] + f_2\left[C^n{}_j - C^n{}_{j-1} + C^{n+1}{}_j - C^{n+1}{}_{j-1}\right] \quad (10)$$

A substitution of eq. (10) and (6) in eq. (5) leads to:

$$w_1 C^{n+1}{}_{j-1} - C^n{}_{j-1} + w_2 C^{n+1}{}_j - C^n{}_j + w_3 C^{n+1}{}_{j+1} - C^n{}_{j+1} =$$

$$\alpha_j \frac{\Delta t}{2\Delta x}\left\{f_1\left[C^n{}_{j+1} - C^n{}_j + C^{n+1}{}_{j+1} - C^{n+1}{}_j\right] + f_2\left[C^n{}_j - C^n{}_{j-1} + C^{n+1}{}_j - C^{n+1}{}_{j-1}\right]\right\}$$

Let's define the dimensionless parameter $\alpha_{D_j} = \alpha_j \frac{\Delta t}{2\Delta x}$ (11)

Rearranging gives:

$$(w_1 + f_2\alpha_{D_j})C^{n+1}{}_{j-1} + (w_2 + (f_1 - f_2)\alpha_{D_j})C^{n+1}{}_j + (w_3 - f_1\alpha_{D_j})C^{n+1}{}_{j+1} =$$

$$(w_1 - f_2\alpha_{D_j})C^n{}_{j-1} + (w_2 - (f_1 - f_2)\alpha_{D_j})C^n{}_j + (w_3 + f_1\alpha_{D_j})C^n{}_{j+1} \quad \text{or}$$

$$aC^{n+1}{}_{j-1} + bC^{n+1}{}_j + cC^{n+1}{}_{j+1} = d \quad (12)$$

where:

$$a = w_1 + f_2\alpha_{D_j}, \quad b = w_2 + (f_1 - f_2)\alpha_{D_j}, \quad c = w_3 - f_1\alpha_{D_j}, \quad p = w_1 - f_2\alpha_{D_j}, \quad q = w_2 - (f_1 - f_2)\alpha_{D_j},$$
$$r = w_3 + f_1\alpha_{D_j}, \quad d = pC^n{}_{j-1} + qC^n{}_j + rC^n{}_{j+1} \quad \text{.....(12.1-12.7)}$$

Eqs. (12) to (12.7) gives the concentration at the three elements after one timestep. This system can be expressed as a tridiagonal matrix and can be solved with Thomas algorithm.

$$\begin{bmatrix} b_1 & c_1 & & & & & & & & \\ a_2 & b_2 & c_2 & & & & & & & \\ & a_3 & b_3 & c_3 & & & & & & \\ & & . & . & . & & & & & \\ & & & . & . & c_j & & & & \\ & & & & a_j & b_j & . & & & \\ & & & & & . & . & . & & \\ & & & & & & a_{N-3} & b_{N-3} & c_{N-3} & \\ & & & & & & & a_{N-2} & b_{N-2} & c_{N-2} \\ & & & & & & & & a_{N-1} & b_{N-1} \end{bmatrix} \begin{bmatrix} C^{n+1}{}_1 \\ C^{n+1}{}_2 \\ C^{n+1}{}_3 \\ \\ \\ \\ \\ \\ \\ C^{n+1}{}_{N-1} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \\ \\ \\ \\ d_{N-3} \\ d_{N-2} \\ d_{N-1} \end{bmatrix} \quad \text{.....(13)}$$

The initial conditions should be given for the first timestep. The concentration at the last element (bottom) must be also known, that is, the boundary condition. An appropriate equation should be written for the first element:

$at_{j=1} \Rightarrow$

$a = 0,\ b = w_2 + \alpha_{D_1},\ c = w_3 - \alpha_{D_1},\ p = 0,\ q = w_2 - \alpha_{D_1},\ r = w_3 + \alpha_{D_1},\ d = qC^n{}_1 + rC^n{}_2$

Another appropriate equation should be written for the element before the last one:

$at_{j=N-1} \Rightarrow$

$a = w_1 + f_2\alpha_{D_{N-1}},\ b = w_2 + (f_1 - f_2)\alpha_{D_{N-1}},\ c = 0,$

$p = w_1 - f_2\alpha_{D_{N-1}},\ q = w_2 - (f_1 - f_2)\alpha_{D_{N-1}},\ r = w_3 + f_1\alpha_{D_{N-1}},$

$d' = pC^n{}_{N-2} + qC^n{}_{N-1} + rC^n{}_N,\ d = d' - (w_3 - f_1\alpha_{D_{N-1}})C^n{}_N$

Slip Phenomena Treatment

The velocity of the particles depends on the velocity of the liquid and the slip velocity between the liquid and the particles,

$$V = \overline{Va} - Vs \quad \text{.....(14)}$$

where V is the velocity of the particles, $\overline{Va}$ is the average velocity of the liquid, and Vs is the slip velocity. Because the cross-sectional area of liquid flow depends on the concentration, $\overline{Va}$ also depends on the concentration:

$$\overline{Va}_j = \frac{Q}{A\left(1 - C_j\right)} \quad \text{.....(15)}$$

where Q is the liquid flow rate. In this model, $\overline{Va}$ is assumed constant.

Solving Stock Law, which relates the drag force to the gravity and buoyancy forces, for slip velocity, gives the following equation:

$$Vs_j = \sqrt{\frac{4gdp\left(\rho_p - \rho_l\right)}{3\rho_l K_{D_j}}} \quad \text{.....(16)}$$

It has been shown that the drag coefficient K_{D_j} depends on the Reynolds number, particles shape, and particles concentration. In this model the particles are assumed to be spheres. Ariza, using dimensionless analysis with experimental data, derived a relationship between the drag coefficient and the Reynolds number for a single particle.

$$K_{Do} = \frac{24}{\mathrm{Re}_o}\left(1 + 0.15\mathrm{Re}_o^{0.687}\right) \Leftrightarrow \mathrm{Re}_o < 1000 \quad (17)$$

where the Reynolds number for a single particle of a non Newtonian fluid is define:

$$\mathrm{Re}_o = \frac{\rho_l Vs_o^{2-n'} d_p^{n'}}{k'} \quad (18)$$

As can be seen, both eqs. (16) and (17) depends on the slip velocity and must be solved iteratively.

A simpler relationship can be used as a first approximation, $K_{Do} = \frac{40}{\mathrm{Re}_o} + 1$ (19)

Substitution of this eq. in eq. (16) for a single particle, that is, $Vs_o = \sqrt{\frac{4gdp\left(\rho_p - \rho_l\right)}{3\rho_l K_{Do}}}$, applied to a Newtonian fluid, leads to the second order equation: $\frac{\rho_l d_p}{\mu} Vs_o^2 + 40Vs_o - \frac{4gdp^2\left(\rho_p - \rho_l\right)}{3\mu} = 0$, which can be solved as:

$$Vs_o = \frac{-40 + \sqrt{40^2 - 4a'c'}}{2a'} \quad (20)$$

where the coefficients are, $a' = \frac{\rho_l d_p}{\mu}$, and $c' = -\frac{4gdp^2\left(\rho_p - \rho_l\right)}{3\mu}$

The use of eq. (20) gives a starting point for the iteration with eqs. (16) and (17) mentioned above, which gets the slip velocity and Reynolds number for a single particle flowing through a non Newtonian fluid. Convergence is obtained in few trials.

A relationship between the multi-particle drag coefficient and the single particle drag coefficient was also obtained by Ariza as a function of the concentration.

$$K_{D_j} = \frac{K_{Do}}{\left(1 - C_j\right)^{2(m-1)}} \quad (21)$$

where m was experimentally obtained $m = \frac{4.7\left(1 + 0.15\mathrm{Re}_o^{0.687}\right)}{1 + 0.253\mathrm{Re}_o^{0.687}}$ (22)

Now, the velocity of the particles can be calculated with eq.(14) substituting eq. (21) in eq. (16):

$$V_j = \overline{Va} - \left(\frac{4gdp\left(\rho_p - \rho_l\right)}{3\rho_l}\frac{\left(1 - C_j\right)^{2(m-1)}}{K_{Do}}\right)^{1/2} \quad (23)$$

In order to solve for α of eq. (5), not only the velocity of particles V is needed, but also its derivative respect to the concentration of particles $\frac{dV}{dC}$. Taking the derivative of eq. (23) with respect to the concentration and rearranging gives:

$$\left(\frac{dV}{dC}\right)_j = \left(\frac{4gdp\left(\rho_p - \rho_l\right)}{3\rho_l K_{Do}}\right)^{1/2}\left(1 - C_j\right)^{m-2}(m-1) \quad (24)$$

where m is given in eq. (22) and K_{Do} is given in eq. (17)

For a given concentration, α can now be determined using eqs. (23) and (24). In the numerical model, α for each element is assumed known, so an iteration process must be used to calculate the concentration along the well for each time step. The value of α used for the finite difference model is the average between the times; i.e., α^n and α^{n+1}, the latter being the result of an iteration process between the algorithm for calculating the tridiagonal matrix and the algorithm for calculating α^{n+1} with the eqs. (4), (23), and (24).

A program that includes all the algorithms mentioned above was made in FORTRAN. Since the program was improved to analyze hole cleaning and shut in problems, it will be described later.

Model Testing and Tuning

In order to check the stability and accuracy of the numerical model, tuning parameters such as time step and weighting factors were analyzed for negligible slip velocity. The analysis of the α algorithm and the slip velocity was also coupled and some of the tuning parameters were readjusted.

Initial Tuning

A constant concentration for a specific time was used as a boundary condition. When no slip, the solution of the system to this boundary condition, as mentioned before, will be a step function of height given by the difference between the initial concentration and the temporal concentration at the boundary. The starting data used for the analysis is given in table 1.

Table. 1. General Data used

g = 980 cm/s	dt = .001 s
dp = 2.E-6 cm	Length = 200. cm
den_p = 2.6 g/cc	Va = 10. cm/s
den_L = 1 g/cc	w1 = 2.E-1
k' = 1E-2 eqP	w3 = 2.E-1
n = 1	f2 = 4.5E-1
Vs_{tol} = 1%	N = 200 blocks
$Alpha_{tol}$ = 1%	T = 20.1 sec

Note that the no slip condition is obtained using a very small particle diameter. This could also be achieved using very high viscosity or assuming particle density equal to liquid density. Also note that the weighting factors are the same, $w_1 = w_3$, which means that eq. (6.1) apply. An initial concentration of zero along the well is set for all runs. The boundary condition is given as a constant concentration of 10% at the bottom (deepest element) for a period of two seconds and 0% for the remaining time. This BC's will represent a step function of height 0.1 (concentration), and with the particles velocity being 10 cm/s, the width will be 20-cm long. After 10 seconds, the slug of the moving particles should be located between 100 cm and 120 cm from the top of the well. Note that each block is 1-cm length and the block number corresponds also to cm from the top.

The optimum distance increment and time step are related to the equation $\left(\frac{dx}{dt}\right)_C = V$. For this case, the time step dt obtained using this equation is 0.1 seconds. Stone&Brian suggest using less than half of the calculated dt.
Fig. 3 shows the result of the program using different timesteps for the set of data given.

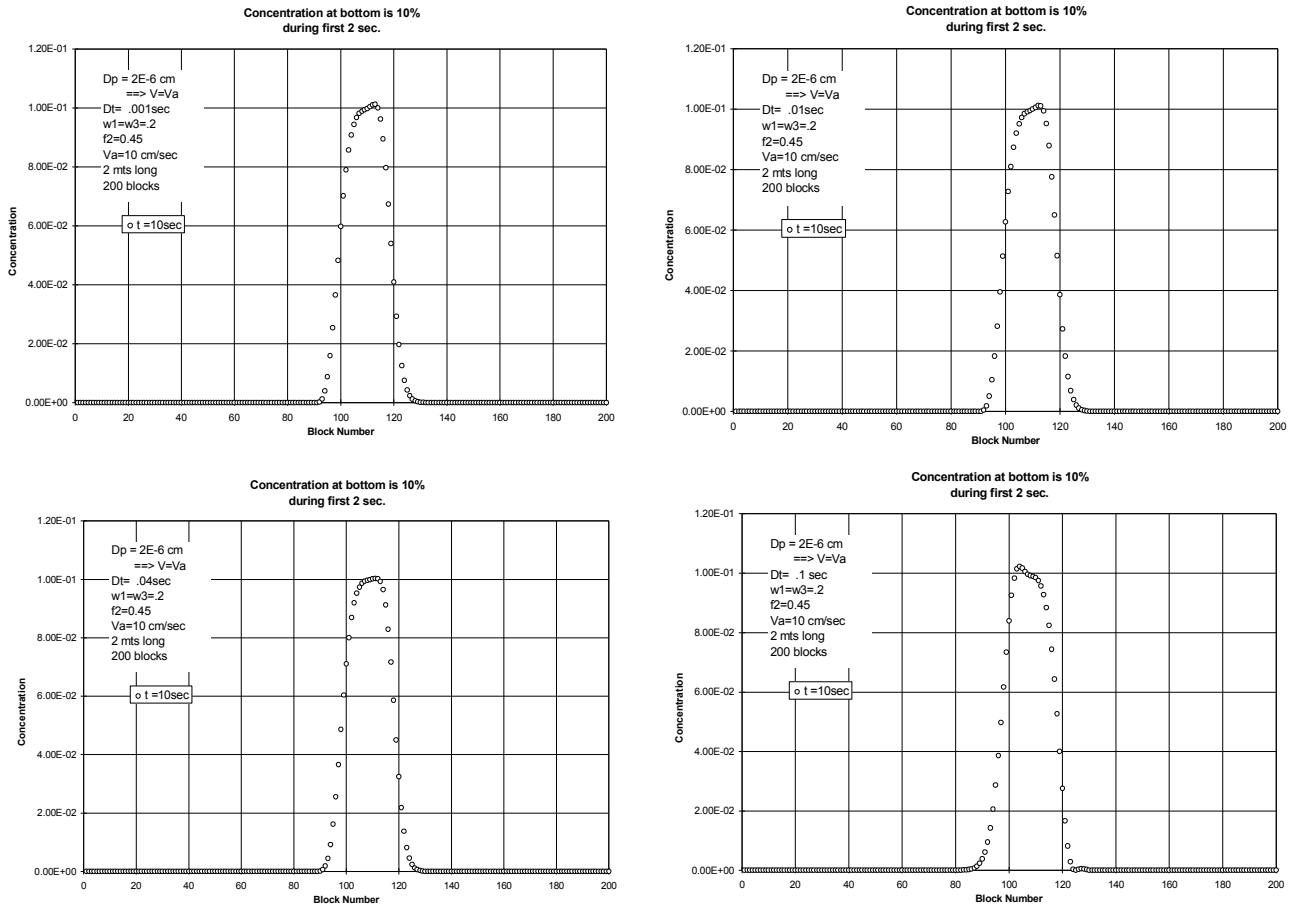

Fig. 3-Time step Analysis

The results of 0.001s, 0.01s, and 0.04s are nearly the same. A time step of 0.04s is chosen as the optimum considering the computer time.

The effects of the weighting factor f2 (see eq. 8) were analyzed and are shown in Fig. 4. High oscillations are obtained when f2 is set 0.5 (see eq. 8.1). The best model performance is chosen to be one with f2 of 0.45.

Note that $w_1 = w_3$ held for this runs, which means that eq. (6.1) apply. Recall $\theta = w_1 + w_3$, then $\theta = 2w_1$. The parameter θ was analyzed and the result is shown in Fig. 5.

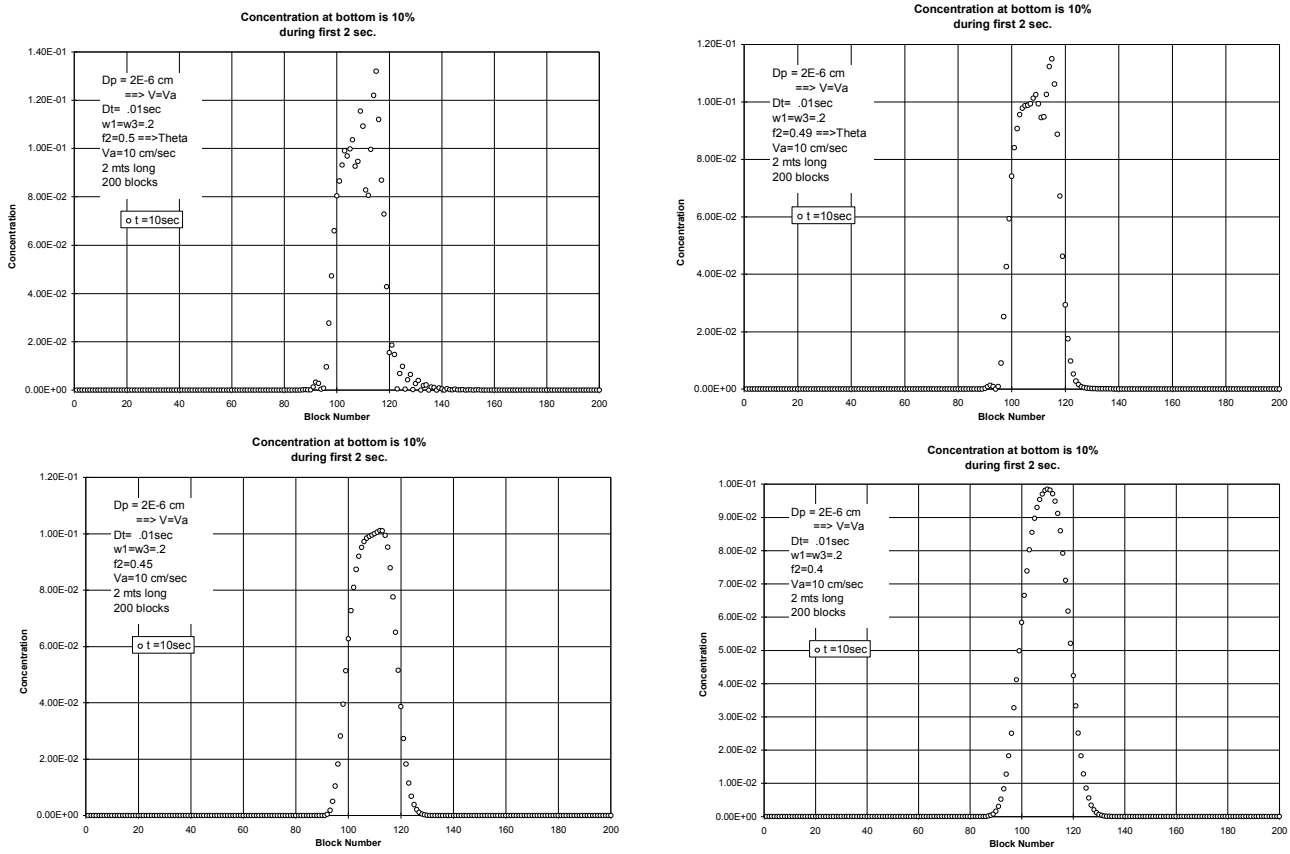

Fig. 4-Analysis of weighting factor f2

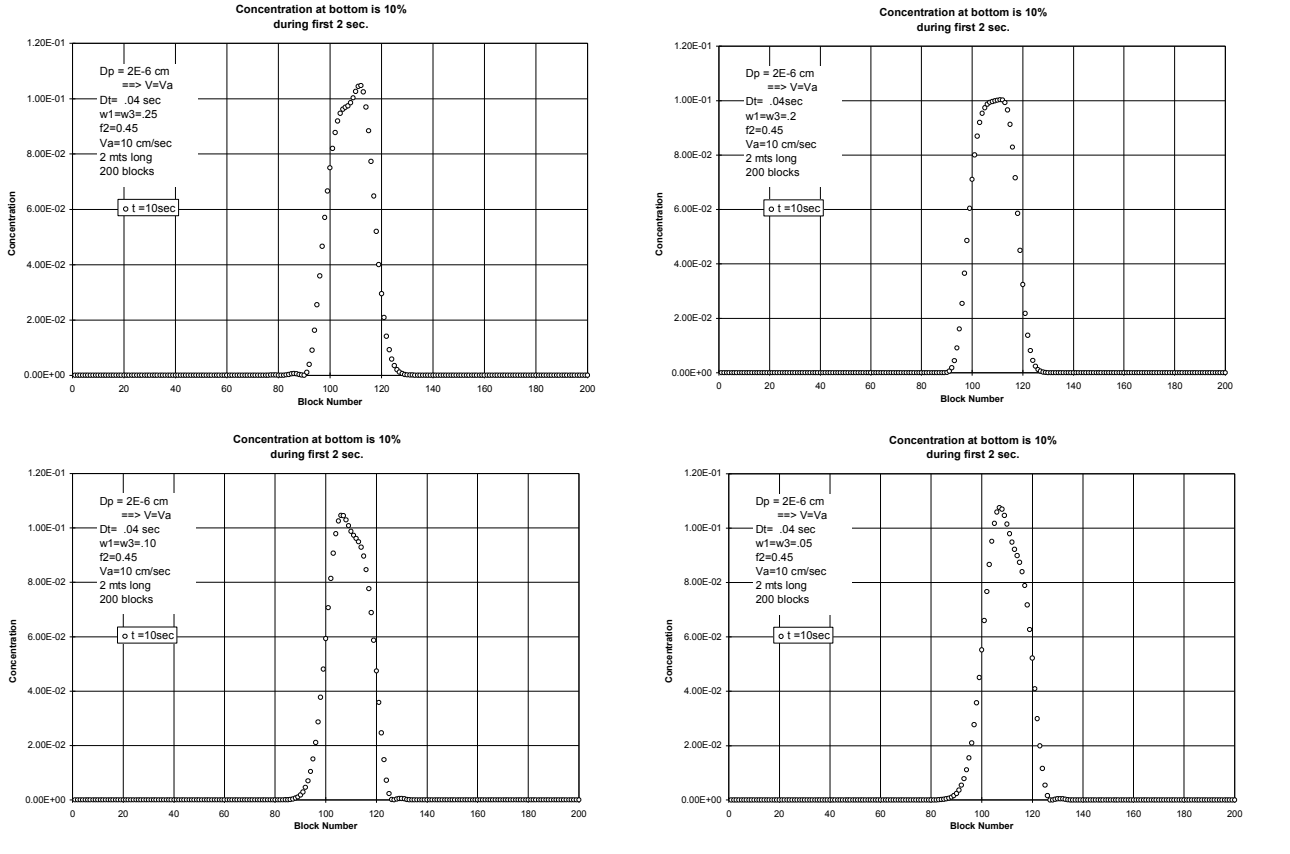

Fig. 5-Analysis of Parameter θ

The best performance is achieved for θ close to 0.4, $w_1 = w_3 = 0.2$.

An analysis of the weighting factors defined in eq. (6), w_1 and w_3, was performed and is shown in Fig. 6a. Using Higher values of w_3 than w_1 gives instability in the back of the slug if the θ is higher than 0.4.

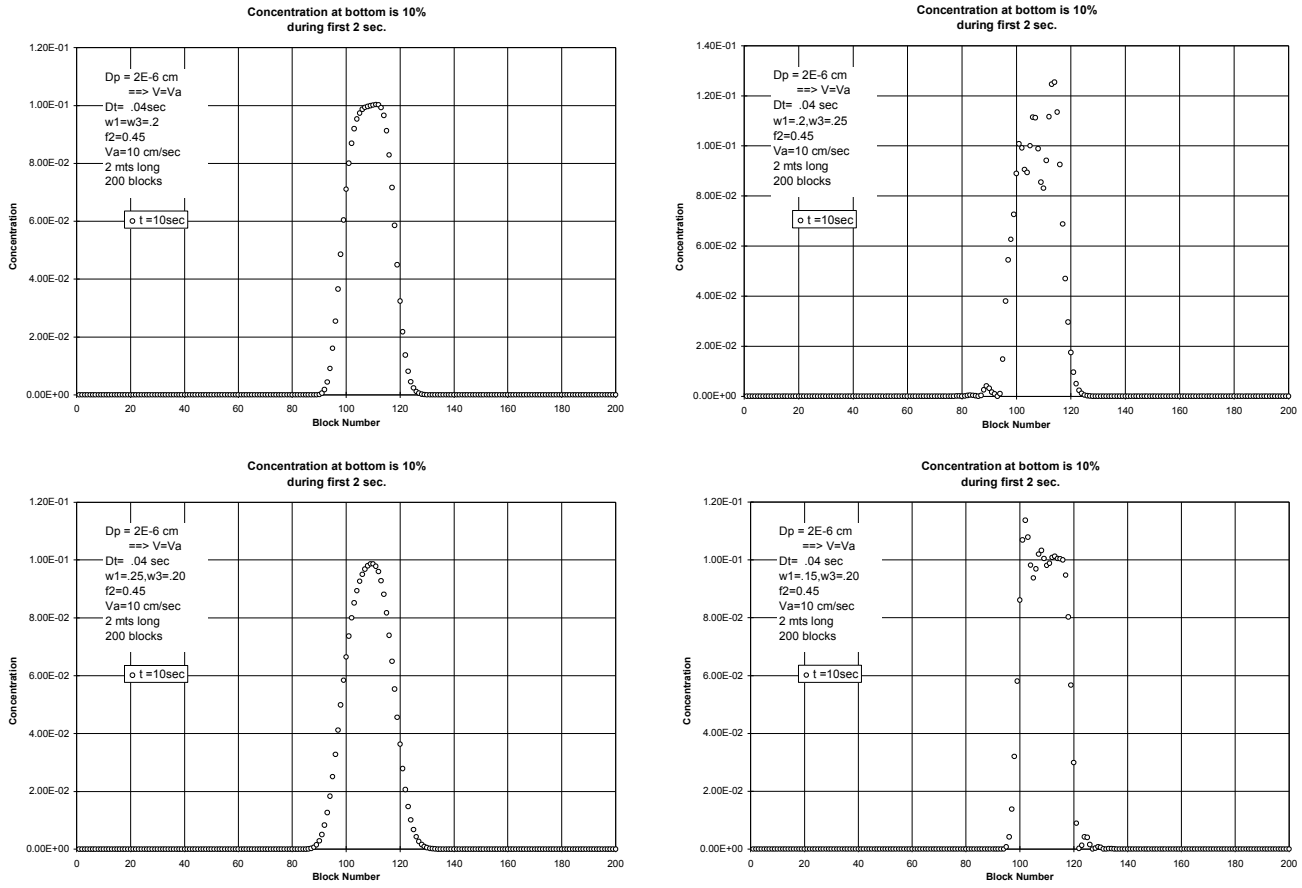

Fig. 6.A- Analysis of weighting factors w_1 and w_3.

Using Higher values of w_3 than w_1 gives instability in the front of the slug if the θ is lower than 0.4. Using higher values of w_1 than w_3 gives stability but the accuracy is diminished. Several runs were made for different values of w_1 and w_3 to obtain the best performance, see Fig. 6.b for results. Values of w_1 = 0.17 and w_3 = 0.20 were found to be the optimum.

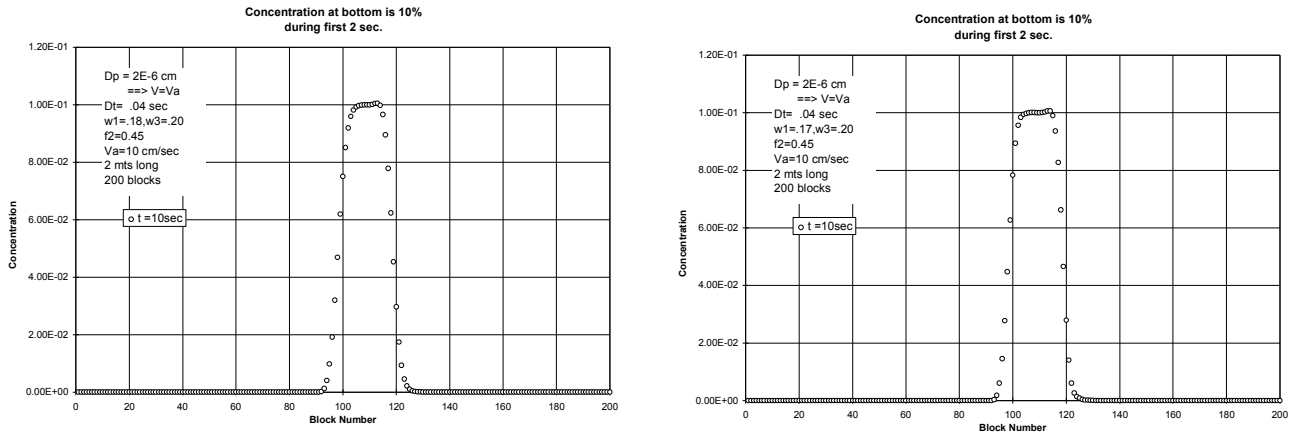

Fig. 6.B. Analysis of weighting factors w_1 and w_3

The slug was verified to be stable and accurate after 18 seconds, just before leaving the well. The slug has to be between 20 cm and 40 cm from the top of the well after 18 sec. See Fig. 6.c.

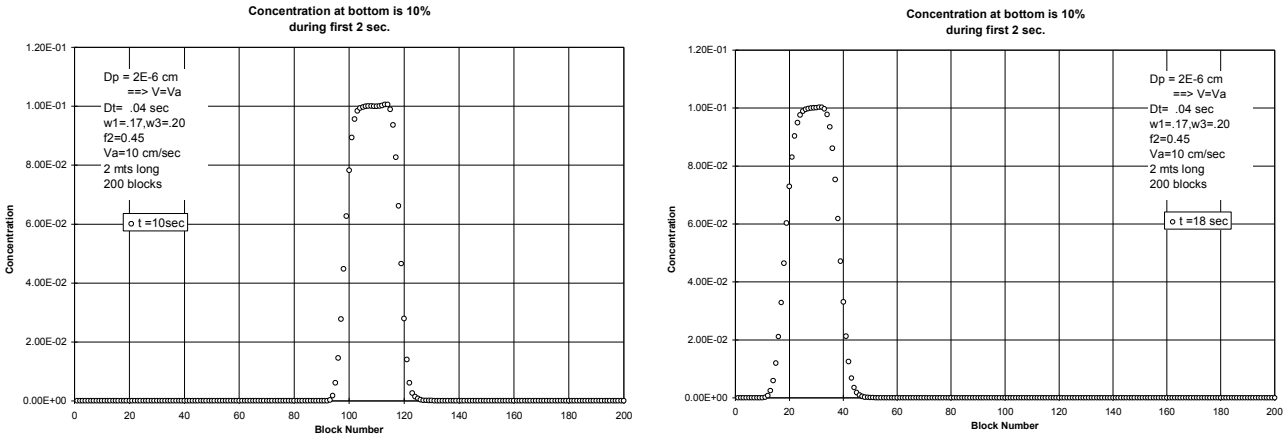

Fig. 6.c- Slug of 10% concentration after 10 and 18 seconds, optimum tuning.

Effects of Slip Velocity

The analysis of the α algorithm and the slip velocity was performed with the same data used for the initial tuning, but particle size was increased to reasonable values. The initial concentration was set to 0%. Instead of the short step function used before, a constant concentration of 5% was used as a boundary condition for any time after the initial. The solution of the system, now dependent on slip, can not be obtained without the model. Fig. 7 shows the result of the model to these conditions for different particle sizes after 10 sec and 19 sec.

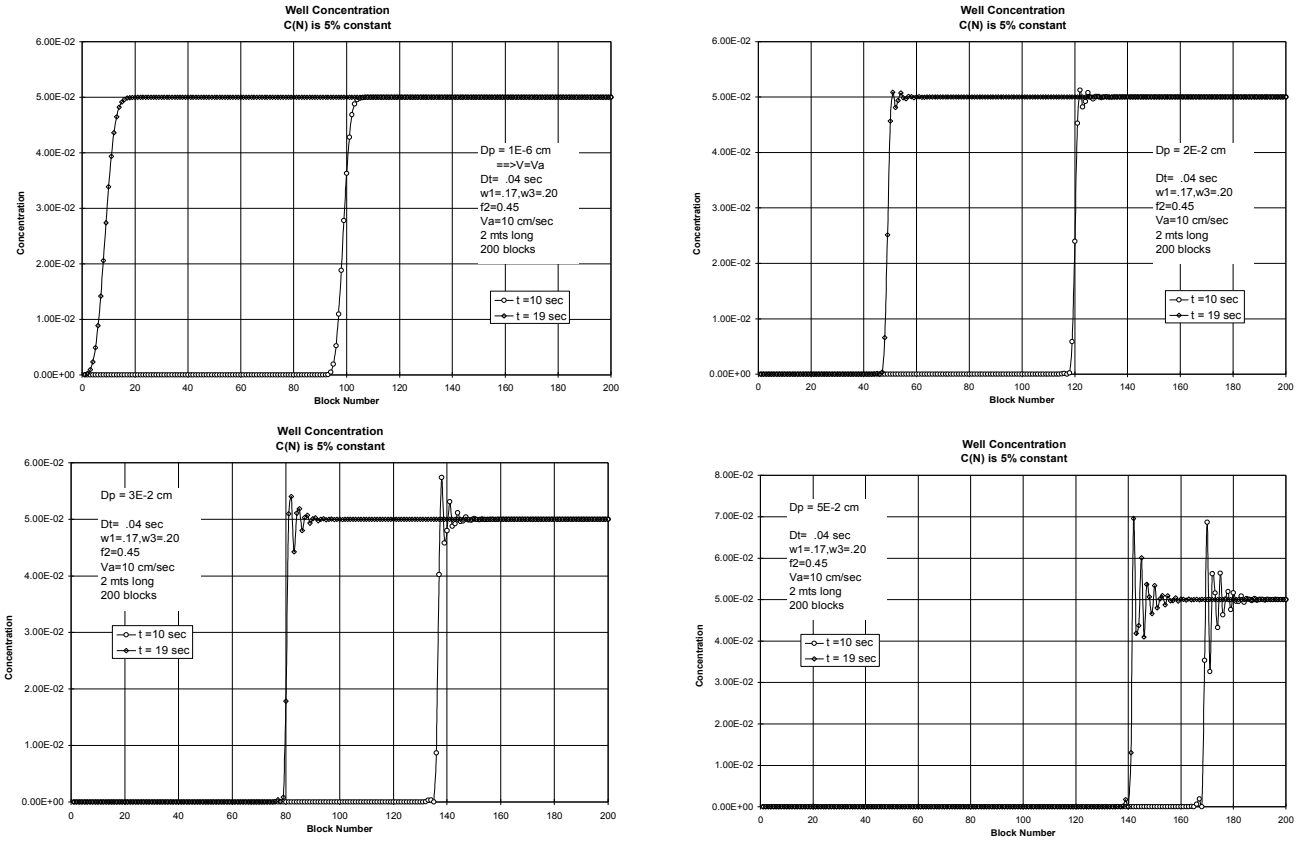

Fig. 7-Effects of Particle Size

When particle size is increased, the slope of the front (slug) is greater, and the solution tends to oscillate. The instability was tried to diminish with the tuning parameters. Where Fig. 8. shows how time step is not affecting the stability, Fig. 9. shows how the weighting factor f2 can eliminate it. And optimum f2 of 0.4 will be used for future runs.

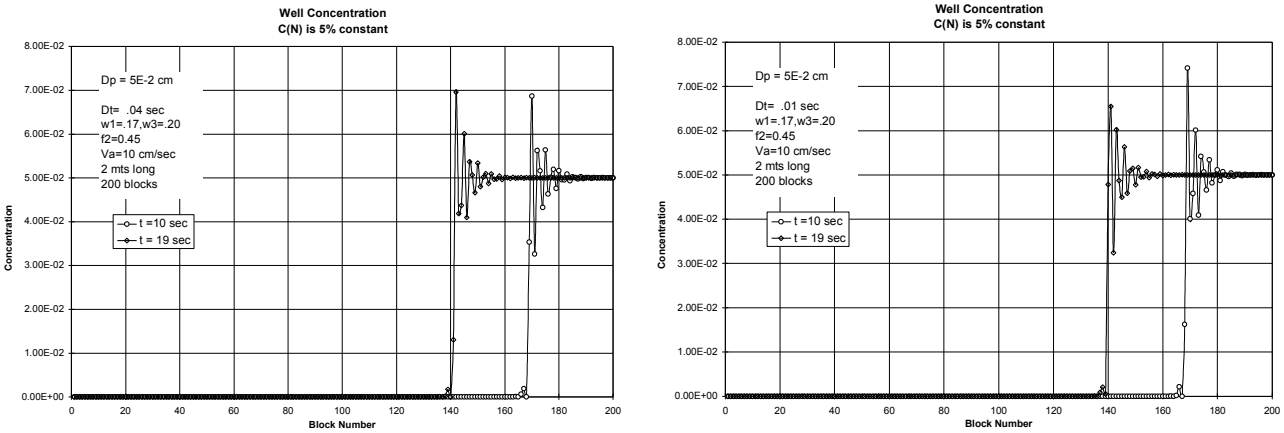

Fig. 8-Effects of timestep

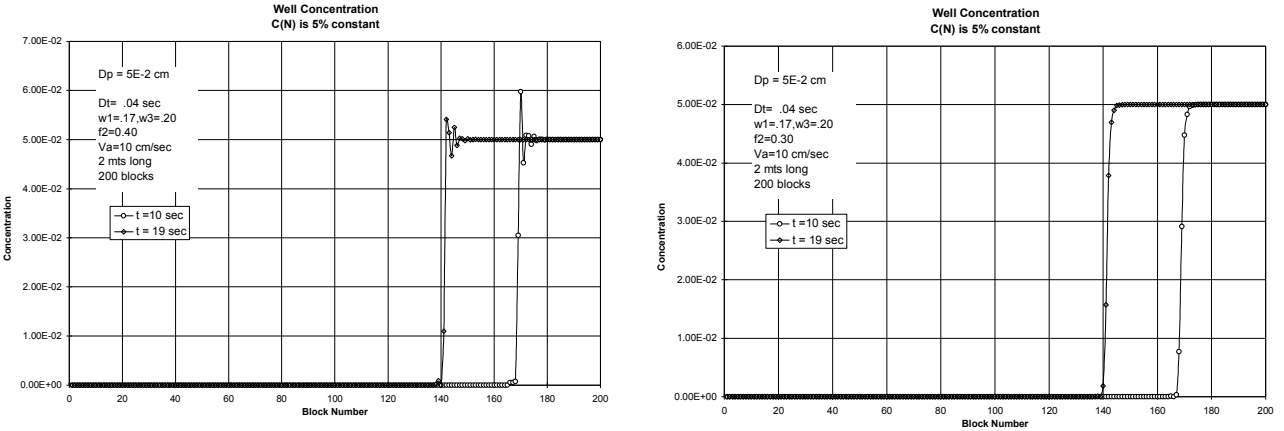

Fig. 9-Effects of weighting factor f2

Effects of Boundary Condition Function

In order to verify that the instabilities seen so far when considering slip are caused by the step function, an analysis of the boundary condition function was performed. An idealized smooth function was used as a temporal boundary condition: a half sine function with peak 10% at 2 sec (4 sec long). Fig. 10 compares the result of the model for both boundary condition functions. It can be noticed that the particles tend to accumulate in the front of the slug and get spread in the back as they go to the surface.

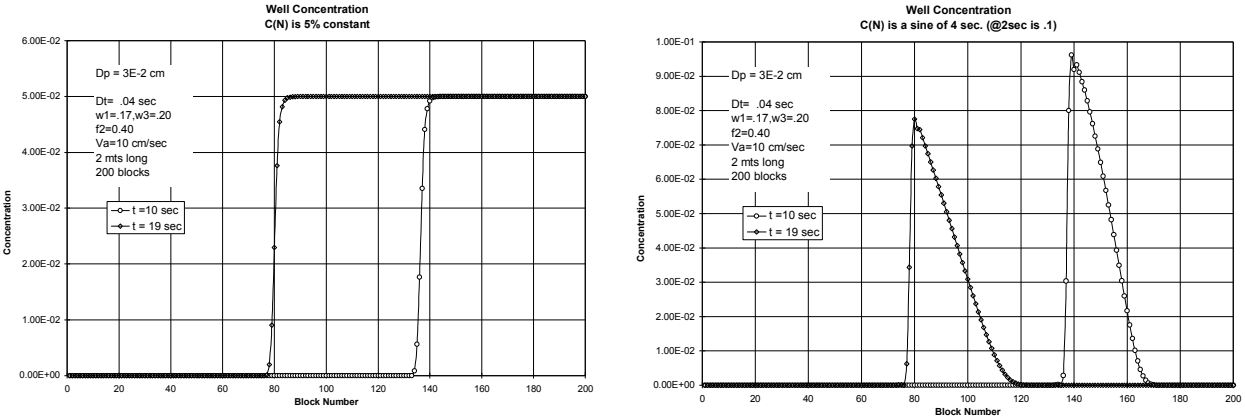

Fig. 10-Result of the model for two different boundary condition functions.

Further analyses of the weighting factors were performed. Fig. 11 shows how, although setting the weighting factors and w_3 (left), and f2 and w_3 (right) to zero, the boundary condition function set for the concentration at the bottom changes its shape while flowing through the well.

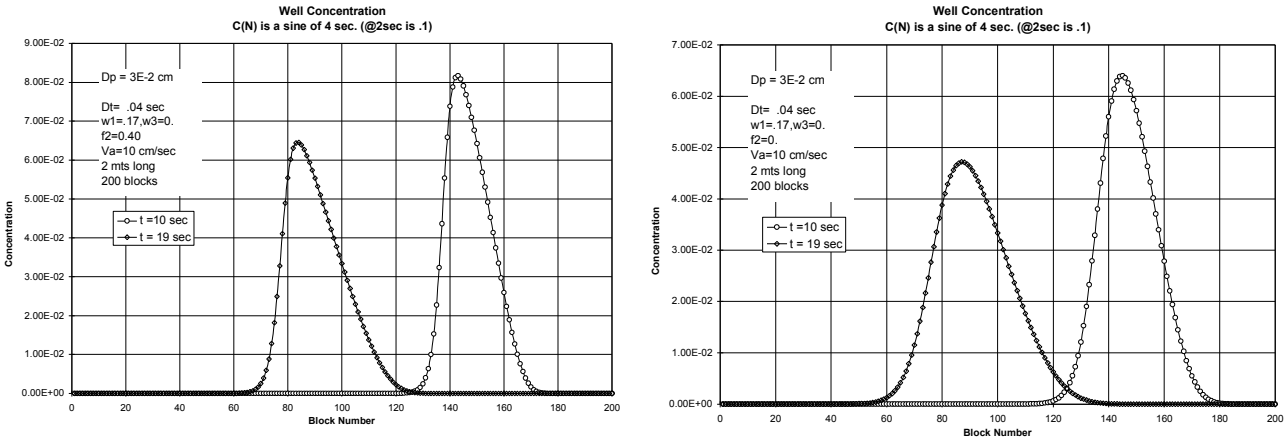

Fig. 11-Effects of weighting factors when slip, w_3 =0 (left), and f2 = w_3=0 (right).

This behavior is associated to the slip velocity dependence on the concentration. The parameter α defined in eq. (4) is no longer a constant. Such numerical model can only obtain the solution to this problem. Fig. 12 compares the results of the model for "the optimum tuning parameters" to the results when neglecting slip using those parameters.

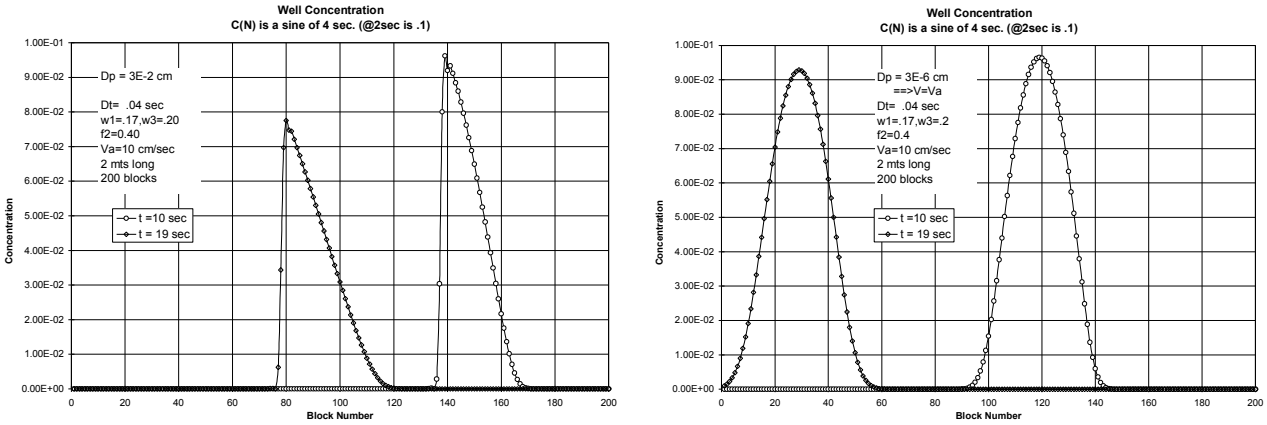

Fig. 12-Comparison of results when neglecting slip.

Coupling Moving Boundary Model

The model becomes more interesting when the analysis of particles along the well is coupled to problems such as well-bore cleaning, and the stack particles in the bottom-hole when the well is shut down. In order to analyze the flow of particles along the well when the bottom-hole is a moving boundary defined by the height of stack particles, mass balance in the stack section is again the right equation to apply.

$$\frac{d}{dt}\int \rho d\nu + \sum_{\substack{control \\ surface}} \rho \overline{V} \circ \overline{A} = 0 \qquad (25)$$

The first term represents the rate of accumulation, and the second, represents the net flow. The parameter $\nu = (1-\phi)Ah$, is the volume of particles. The parameter ϕ is the void fraction in the stack zone. If density of particles is assumed constant as before, the equation becomes:

$\frac{d\nu}{dt} - V_p AC = 0$, where

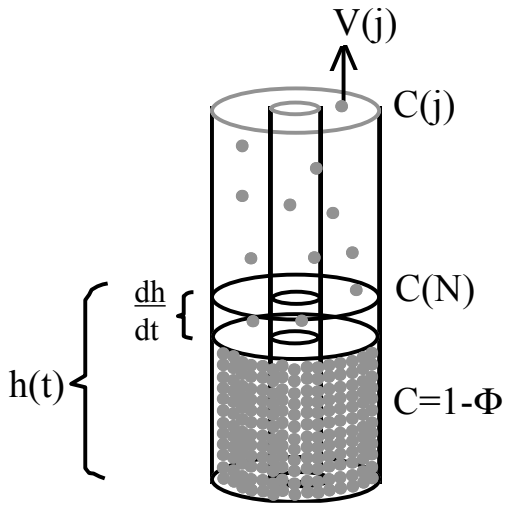

Fig. 13-Diagram for bottom-hole mass balance

The first term represents the rate of accumulation, and the second, represents the net flow. The parameter $\nu = (1-\phi)Ah$, is the volume of particles. The parameter ϕ is the void fraction in the stack zone. If density of particles is assumed constant as before, the equation becomes:

$$\frac{d\nu}{dt} - V_p AC = 0 \quad \text{.....(26)}$$

where V_p is the velocity of particles seen by the interface of stack particles:

$$V_p = -V_N + \frac{dh}{dt} \quad \text{.....(27)}$$

where V_N is the velocity of the particles in the deepest element. Recall $V_N = \overline{Va}_N - Vs_N$.

whereas $\overline{Va}_N$ is be the velocity of the fluid in the element *N*, being zero for the shut down problem.

Assuming $\overline{Va}_N$ constant eq. (26) can be written $\frac{d}{dt}\left[(1-\phi)Ah\right] = \left(-V_N + \frac{dh}{dt}\right)AC_N$. Rearranging gives:

$$\frac{dh}{dt} = \frac{-V_N C_N}{(1-\phi-C_N)} \quad \text{.....(28)}$$

The term $\frac{dh}{dt}$ represents the variation of the height of the stack interface after one timestep. The phenomena represented by eq. (28) can be coupled as a boundary condition to the numerical model for the concentration of particles along the well (NMCPAW) described in last chapter. This can be achieved in two different ways. One could be calculating the timestep of NMCPAW with eq. (28) to clean or accumulate one block of particles, and increase or decrease the number of blocks by one. This is not recommended because it will obviously affect the numerical performance of the NMCPAW, (recall $\left(\frac{dx}{dt}\right)_C = V$). The other way is to calculate how much distance Δh^{n+1} the interface of stack particles changes in one timestep. This distance is also the change of the length L^{n+1} of the NMCPAW.

$$L^{n+1} = L^n - \Delta h^{n+1} \quad \text{.....(29)}$$

The element size will now change for each timestep since the number of blocks are the same:

$$\Delta x^{n+1} = L^{n+1} / N \quad \text{.....(30)}$$

The height of stack particles h^{n+1}, as well as L^{n+1}, will increase or decrease depending if the well is being cleaned out or if it was shut in.

$$h^{n+1} = h^n + \Delta h^{n+1} \quad \text{.....(31)}$$

The algorithm for such a system is shown and self-explained in Figs. 14 and 15.

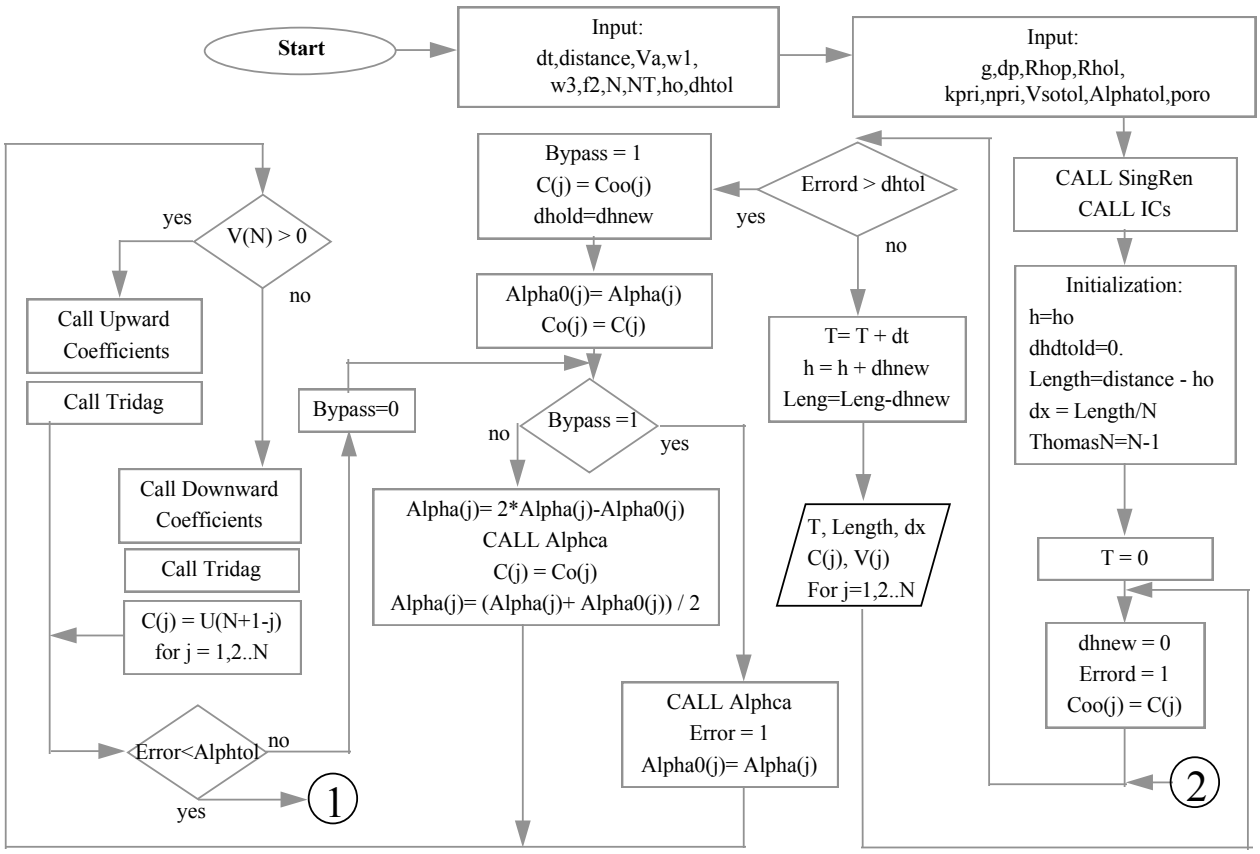

Fig. 14-Flow chart of the computer program

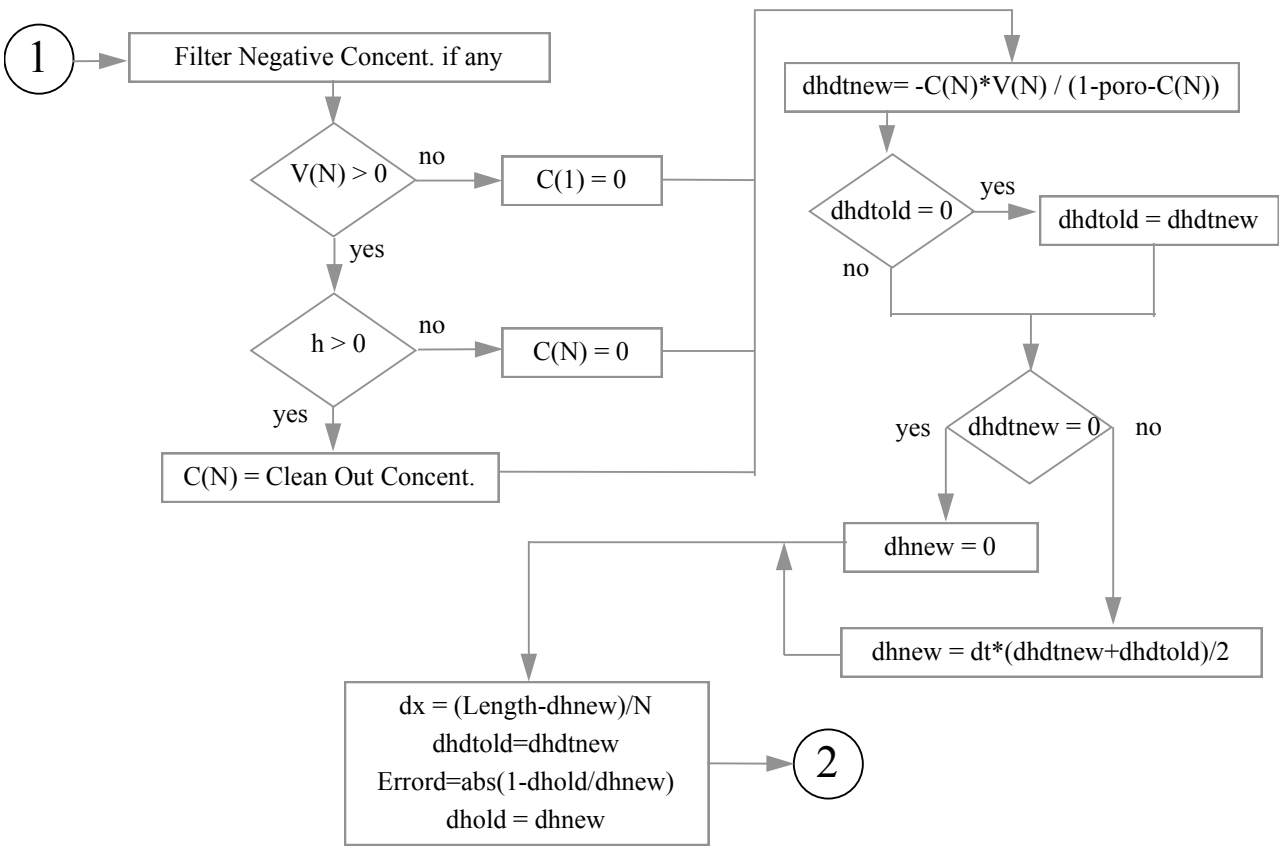

Fig. 15-Flow chart of the computer program.

Discussion of Results

Shut Down Analysis

For the problem of analyzing the concentration of particles when the well is shut down the program was run with a similar data set as before using "the optimum tuning parameters". Two different particle sizes were analyzed. The initial concentration was set 1% along the well. The bottom-hole of the well was supposed to be cleaned at the beginning (no stack). Fig. 16.a shows two plots with the results of the simulation. The one at the left shows the concentration in the last element and the height of stack particles as a function of time. The plot at the right shows the concentration along the well at four different times. As can be seen, the particles accumulate in the bottom and the rest of the well is already cleaned after about seven seconds for that particle size flowing through the specified fluid.

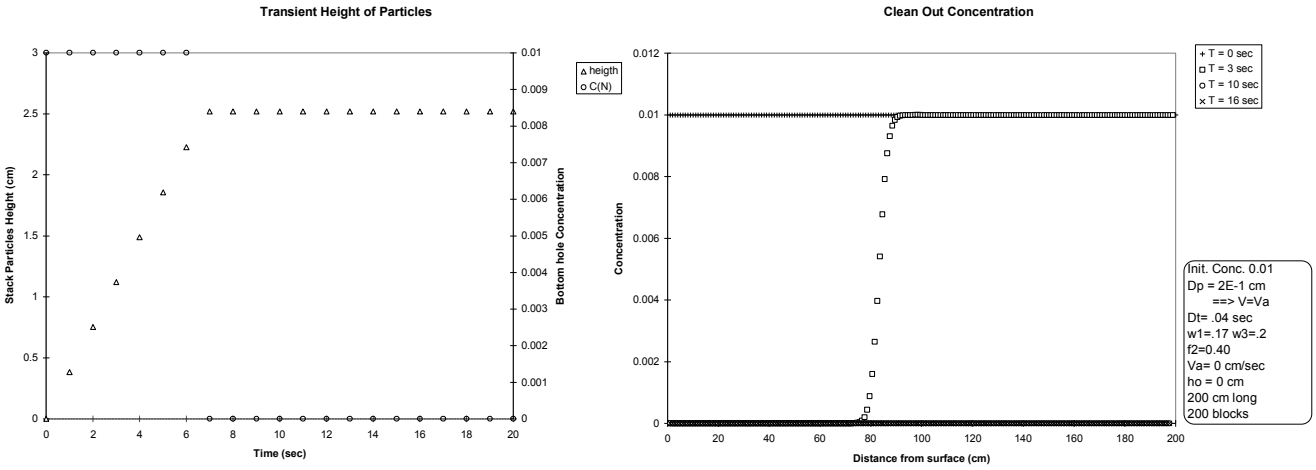

Fig. 16.A- Shut Down Analysis

Fig. 16.b shows similar plots with the results of the model for as much as half the particle size of the Fig. 16.a. The particles accumulate in the bottom and the rest of the well is cleaned after about 13 seconds.

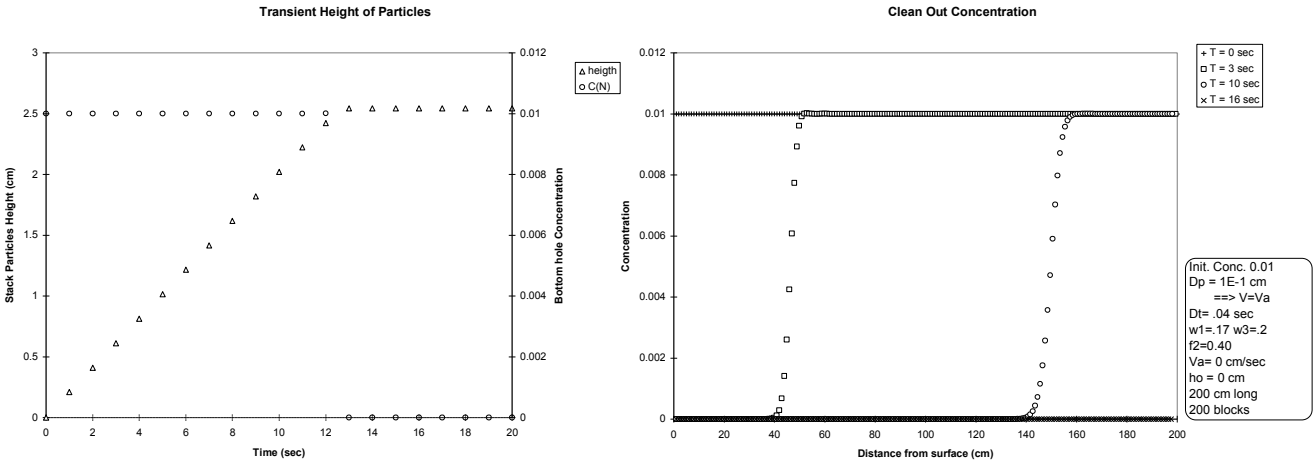

Fig. 16.B- Shut Down Analysis

Clean Out Analysis

Several runs were performed to analyze the clean out problem. Again, the initial concentration was set 1% throughout the well. The same data used in the shut in problem was used for this analysis, but neither the velocity of liquid, nor the initial stack height of particles was assumed zero. No slip is considered since the particle size was set close to zero. Fig. 17 shows the results when the initial stack height of particles was 20 cm. Fig. 18 shows the results when the initial stack height of particles was 10 cm. It can be noticed in both figures how the particles stack at the bottom in the beginning of the simulation will create a slug that travels along the well while the bottom is being cleaned.

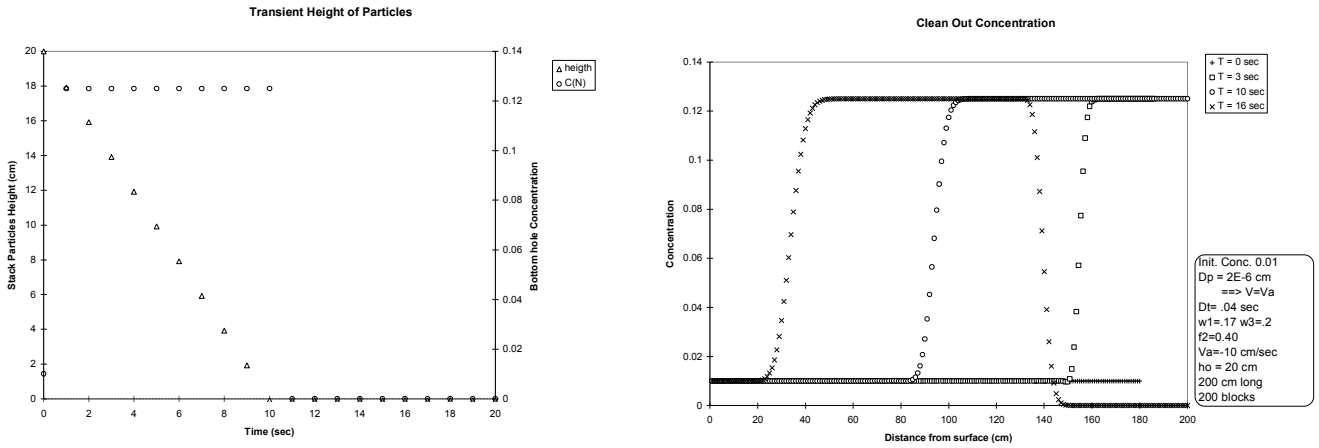

Fig. 17-Clean Out Analysis. Initial stack height of particles was 20 cm.

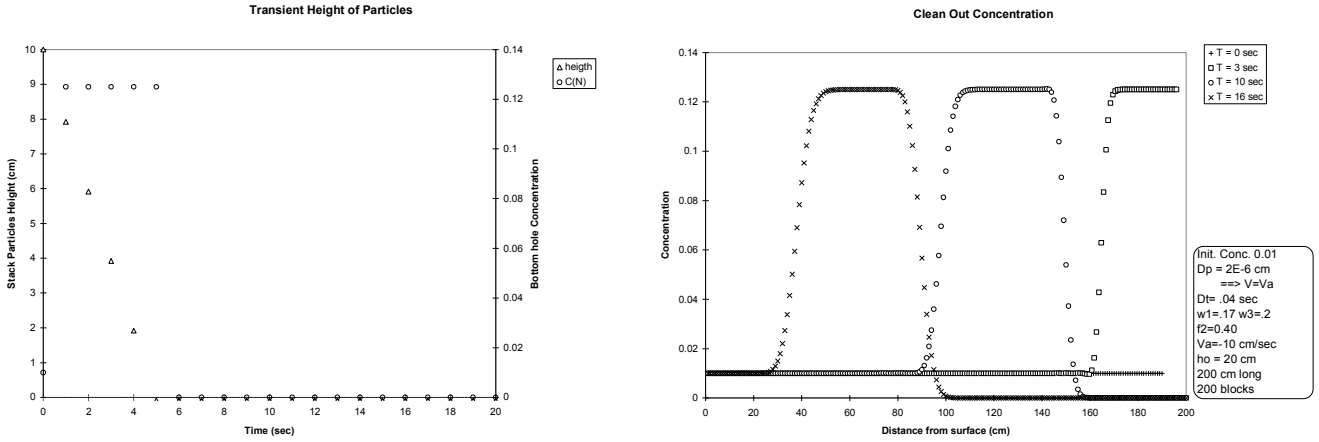

Fig. 18-Clean Out Analysis. Initial stack height of particles was 10 cm.

Fig. 19 shows the results with the initial stack height of particles of 20 cm but the particle size and the liquid velocity were increased (more slip).

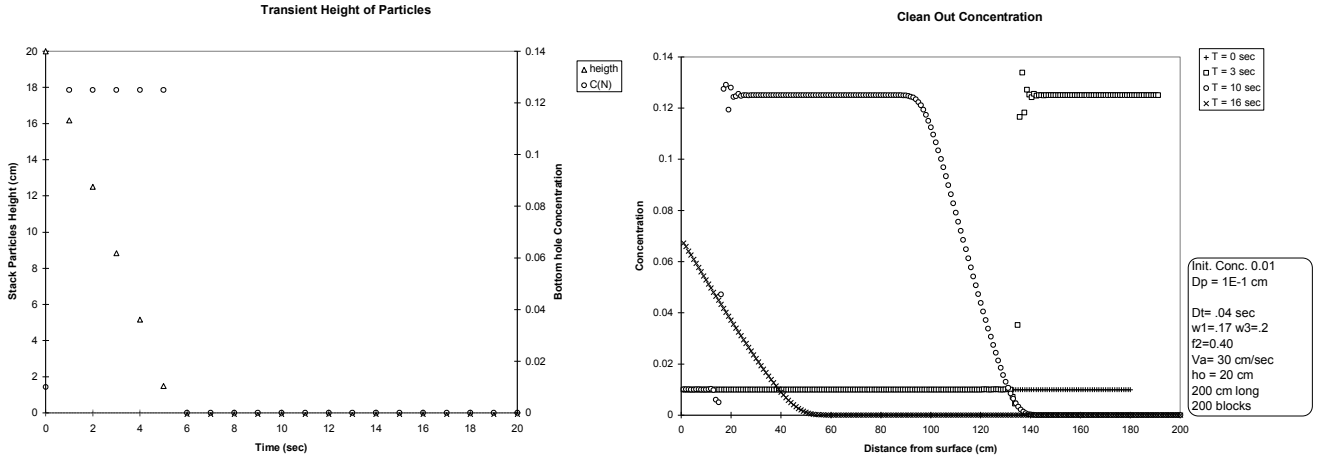

Fig. 19-Clean Out Analysis considering slip. Initial stack height of particles was 20 cm.

The bottom hole concentration when cleaning out was assumed constant for all the runs. This value depends on many factors like the concentration of the stack particles (1-ϕ), the height of stack particles, the liquid velocity, geometry of the well-bore, and some others. As an attempt to simulate a non constant bottom-hole concentration, a sine function dependent on the height and ϕ was analyzed. The function was defined: **C(N)=sin(((1-poro)/6.)*3.1416*h/ho)+0.01.** Fig. 20 shows

the results of the program with such a boundary condition. No slip was considered in this simulation. Fig. 21 shows the same plots but the particle size and the liquid velocity were increased, considering the effect of slip.

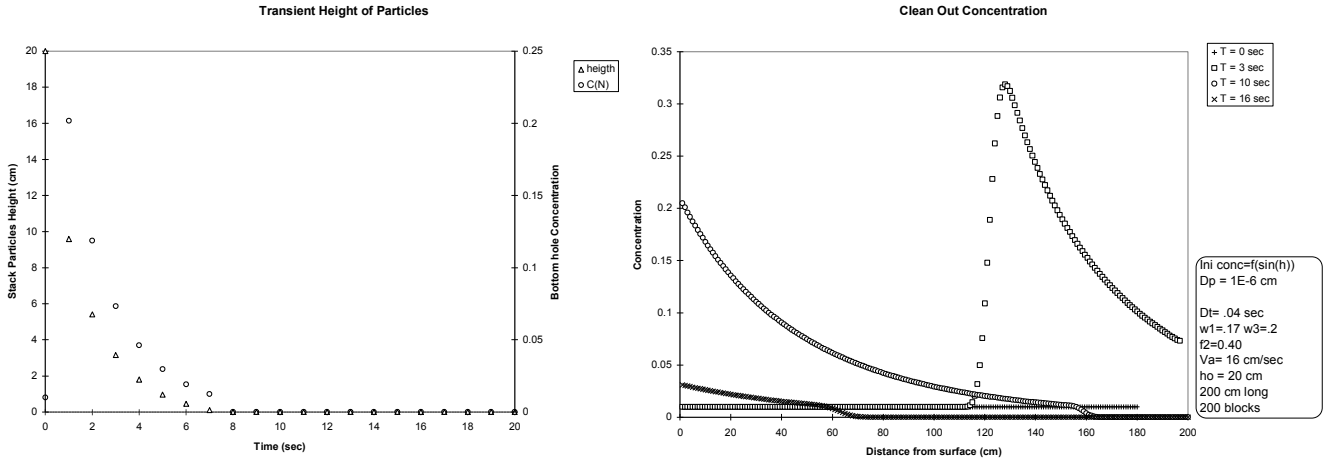

Fig. 20-Clean Out Analysis. ho = 20 cm. Sine function as BC's. No slip.

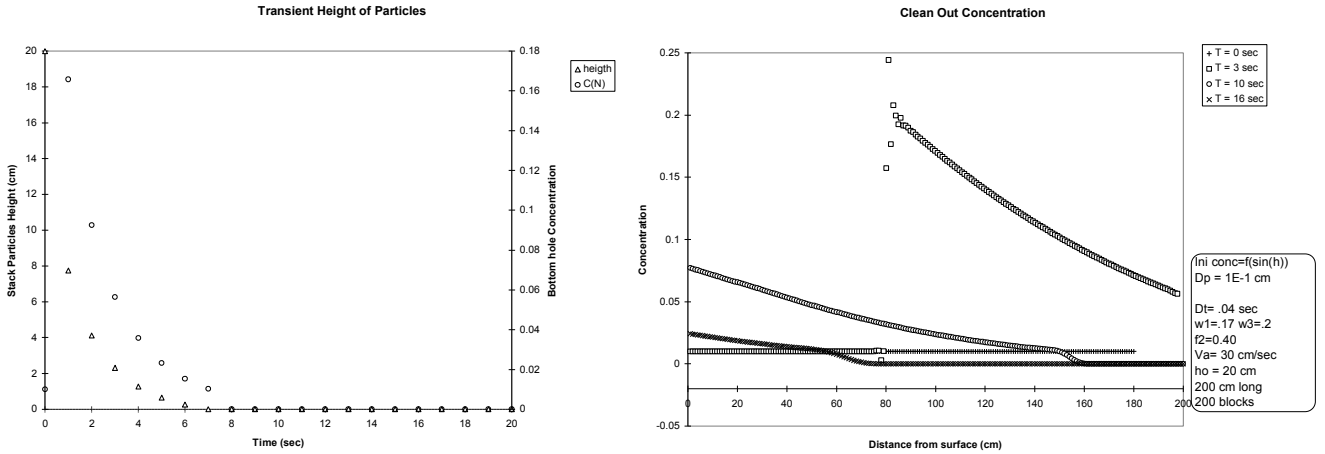

Fig. 21-Clean Out Analysis. ho = 20 cm. Sine function as BC's. Considering slip.

Conclusions

A program to analyze the concentration of particles (cuttings) throughout the well has been devised. The numerical model considers hole-cleaning and shut down problems as moving boundaries. The program has been tested for stability and qualitative behavior, and it seems to give good results. The use of Thomas algorithm in this program for the tridiagonal system, limits the flow of particles within the well to only one direction. In a real case, this is typical, but not always true.

Although the program represents very well an idealized unidimensional system, further analysis has to be made to successfully relate results of the numerical analysis to field applications. The velocity of the particles is not uniform in a cross-sectional area. These phenomena can be noticed in a simple laboratory experiment. In the case of flow through the annulus, as in drilling, the effect of having two walls makes the assumption of uniform particle velocity even worse. An empirical factor that accounts for these phenomena dependent on the fluid type could improve the analogy to field applications. A more sophisticated numerical analysis with cylindrical coordinates can also accomplish the job.

Nomenclature

$\overline{A}$ = average cross-sectional area

dp = particle diameter, cm

F = volume of particle loss to formation, cc

g = gravity acceleration, cm/s^2

h = stacked heigth of particles, cm

K_D = drag coefficient

K_{Do} = single particle drag coefficient

k' = power law consistency index, eqP

L = length of well with moving particles, cm

$_m$ = empirical coefficient [2]

$_{n'}$ = power law exponent

N = number of elements

Q = liquid flow rate, cc/s

Re_o = single particle reynolds number

t = time, sec

Vs = slip particle velocity, cm/s

$\overline{V}$ = average particle velocity, cm/s

V_N = particle velocity at interface, cm/s

$\overline{Va}$ = average liquid velocity, cm/s

V_p = relative particle velocity wrt interface, cm/s

w_1, w_2, w_3, f_1, f_2 = num. sol. weighting factors

x = distance, cm

ϕ = void fraction in packed zone

ρ_l = liquid density, g/cm

ρ_p = particle density, g/cm

Δt = time step, s

Δh = heigth increment in time Δt, cm

Δx = element length, cm

α = dimensionless parameter

ν = volume of particles, cc

Subscripts:

$_j$ = arbitrary element number

$_x$ = element location

$_{x+\Delta x}$ = next element location

^{t} = arbitrary time

$^{t+\Delta t}$ = arbitrary time after timestep

^{n} = *o*ld time

$^{n+1}$ = n*ew* time

$_N$ = location at the bottom

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