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Comparison between neuro-fuzzy and fractal models for permeability prediction

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Abstract We have used different techniques for permeability prediction using porosity core data from one well at the Maracaibo Lake, Venezuela. One of these techniques is statistical and uses neuro-fuzzy concepts. Another has been developed by Pape et al. (Geophysics 64(5):1447–1460, 1999), based on fractal theory and the Kozeny–Carman equations. We have also calculated permeability values using the empirical model obtained in 1949 by Tixier and a simple linear regression between the logarithms of permeability and porosity. We have used 100% of the permeability–porosity data to obtain the predictor equations in each case. The best fit, in terms of the root mean-square error, was obtained with the statistical approach. The results obtained from the fractal model, the Tixier equation or the linear approach do not improve the neuro-fuzzy results. We have also randomly taken 25% of the porosity data to obtain the predictor equations. The increase of the input data density for the neuro-fuzzy approach improves the results, as is expected for a statistical analysis. On the contrary, for the physical model based on the fractal theory, the decrease in the data density could allow reaching the ideal theoretical Kozeny–Carman model, on which are based the fractal equations, and hence, the permeability prediction using these expressions is improved.

Keywords Porosity · Permeability · Neuro-fuzzy · Fractal theory · Prediction · Linear regretion · Empirical · General Pape equation

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1 Introduction

The characterization of oil and gas reservoirs requires the knowledge of petrophysical parameters such as capillarity, porosity, and permeability. The permeability is a very complex parameter: for example, its magnitude may change over several orders of magnitude across a reservoir [1]. Its estimation from well logs and core analysis is one of the most challenging tasks of a reservoir analyst [2]. Several approaches have been employed to its estimation. Tixier in 1949 [3] proposed an empirical equation to calculate the permeability from water saturation (S_{wi}) and porosity (ϕ). Empirical techniques based on well log analysis [3, 4] and on exponential or power-law techniques that relate permeability with porosity [5] have also been developed. Recently, Pape et al. [1] proposed a general relationship that allows calculating permeability from porosity or from the pore-radius distribution. This relationship is based on a fractal model for the structure of a porous medium and on the Kozeny–Carman (KC) equations. Finol et al. [2] have also proposed a rule-based fuzzy model approach for permeability prediction from core data. In this case, the relationship between porosity and permeability was presented by means of fuzzy if–then rules; these rules express an inference mechanism: if a hypothesis is known, then the conclusion can be inferred.

In this work, we have compared different techniques for permeability prediction using core porosity data from one well at the Maracaibo Lake (Venezuela): a fractal approach developed by Pape et al. [1], a neuro-fuzzy (NF) model similar to the one proposed by Finol et al. [2], a linear relationship between the logarithms of permeability and porosity derived from the data, and the Tixier relationship. The influence of the density of the input data has also been analyzed, using 100% of the core data and also 25% of these data randomly taken, to obtain the predictor equations.

2 Data and models

The permeability (k) and porosity (ϕ) core data analyzed in this work belong to the depth interval between 13,250 and 13,740 ft of well PX12, located at the Bloque III, Maracaibo Lake, Venezuela (Fig. 1). This interval comprises units C-455 and C-460 of the Lower Eocene-C. Unit C-455 is a massive sandstone that belongs to the lower sequence of interdistributary channels of the area. This unit contains the main accumulation of reserves in the zone. Unit C-460 comprises mainly clean thick sands. The porosity of these two units in the reservoir ranges from 6% to 18%, with a mean value of 13%.

The core porosity values were measured in a porosimeter based on the Boyle's-law helium-expansion method. This is a standard method for measuring either pore volume or grain volume. It uses Boyle's law to determine the pore volume from the expansion of a know mass of helium into a calibrated sample holder (see, for example, Dandekar [6]). A Gas Permeameter MK-7 was used to determine the permeability of the samples. This type of permeameter determines the permeability of porous solids by forcing a gas, such as air, to flow through the test sample. Measurements of the steady-state flow rate and the corresponding pressures provide the necessary data for calculation of the permeability using Darcy law (see, for example, Dandekar [6]).

The models we have used in this work to calculate permeability values from porosity core data have different theoretical bases. We are mainly interested in the fractal-based model [1, 7] and in the adaptive neural-based fuzzy inference system (ANFIS) [8].

2.1 Fractal theory-based model

This approach is theoretically based on a fractal model for the internal structure of a porous medium. Using the modified KC equation and assuming a multifractal structure to calculate the effective pore ratio and the permeability, Pape et al. [1, 7] have obtained a generalized equation to calculate the permeability as a function of the porosity (ϕ), the cementation factor (m), and a

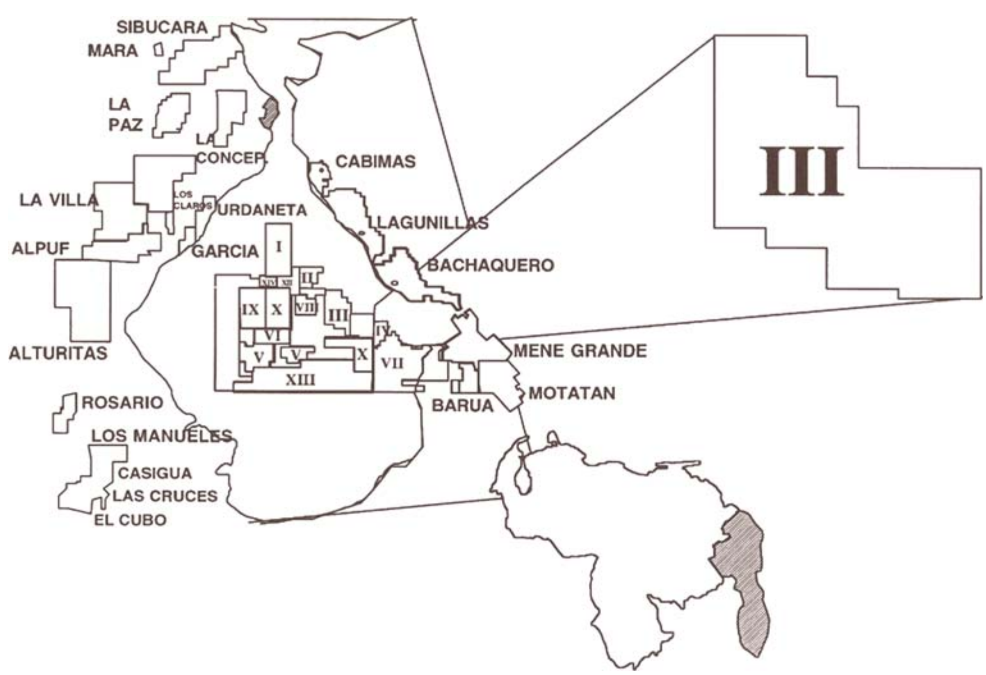

Fig. 1 Geographical setting of well PX12, Maracaibo Lake, Venezuela

fractal dimension (D) associated with the pore-space geometry:

$$k = a\phi + b\phi^{\mathrm{Exp}_1} + c(10\phi)^{\mathrm{Exp}_2} \quad (1)$$

The exponents in Eq. 1 are given by:

$$\mathrm{Exp}_1 = m \quad (2)$$

$$\mathrm{Exp}_2 = m + \frac{2}{c_1(3-D)} \quad (3)$$

where D is the fractal dimension given by:

$$D = 3 + \frac{\log(\phi/0.534)}{0.391\log(1/2\phi)^4} \quad (4)$$

and c_1 depends on the porosity as:

$$c_1 = 0.263\phi^{-0.2};\ 0.39 < c_1 < 1 \quad (5)$$

The parameters a, b, and c depend on the study area and have to be calculated.

Pape et al. [1] have particularized Eq. 1 for different cases. For an average type of sandstone [fractal theory for average type of sandstone (FTAS)], they have obtained:

$$k = 31\phi + 7463\phi^2 + 191(10\phi)^{10} \quad (6)$$

For the Rotliegend sandstone [fractal theory for Rotliegend sandstone (FTRS)], a clean sandstone with relatively large permeability values at any given porosity from northeast Germany, they have obtained:

$$k = 155\phi + 37315\phi^2 + 630(10\phi)^{10} \quad (7)$$

For shaly sandstones [fractal theory for shaly sandstones (FTSS)], they have proposed:

$$k = 6.2\phi + 1493\phi^2 + 58(10\phi)^{10} \quad (8)$$

In this work, we are going to obtain the particularized fractal theory (FT) equation for the study well, and we will compare the results given by this equation with those obtained using Eqs. 6 to 8

2.2 Neuro-fuzzy model

We have used an ANFIS to obtain permeability from porosity core data. ANFIS constructs a fuzzy inference system from a given input/output data set and adjusts the membership function [9] parameters using a backpropagation algorithm [8]. Hence, we have an adaptable hybrid model with five layers that can be interpreted as a neural network with fuzzy parameters or a fuzzy system with distributed parameters. This hybrid NF system is equivalent, under some constrains, to a Takagi–Sugeno–Kang (TSK) model [2, 10]. A TSK system consists of a set of fuzzy if–then rules of the form:

$$R_i: \text{ If } x_1 \text{ is } C_{i1} \text{ and } x_2 \text{ is } C_{i2} \text{ and} \ldots \text{and } x_n \text{ is } C_{in}$$
$$\text{Then } y_i = c_{i1}x_1 + c_{i2}x_2 + +c_{in}x_n + c_{io}$$

In this sense, the output values y_i are considered as linear or constant functions of the input variables x_j. R_i is the ith fuzzy rule; $C_{i1}, \ldots, C_{in}$ are the antecedent linguistic variables and $c_{i1}, c_{i2}, \ldots, c_{in}$ the consequent parameters.

In the ANFIS, each layer has a particular objective [8]:

Layer 1: This layer is composed of n membership functions, each implementing a fuzzy decision rule. Its output is the membership function for which the input variable satisfies the associated C_{ij} term.

Layer 2: This layer computes every possible conjunction of the n decision rules.

Layer 3: This layer normalizes the conjunctive membership functions in order to perceive the inputs.

Layer 4: This layer is a standard perception and associates every membership function with an output (the weights are called consequent parameters).

Layer 5: This layer combines all the individual outputs to obtain the total output (sums evidences).

ANFIS supports a TSK system under the following constrains [8]:

- First-order Sugeno-type systems
- Single output obtained from the weighted average defuzzification
- Unity weigh for each rule

The NF analysis was implemented using the language MatLab (version 6.5) and its toolboxes. To train our NF model, the core porosity values were used as input and the log of the permeability as output. The ANFIS was trained with three gaussian functions (gassmf), 0.5 of tolerance, and 10 epochs were performed. The grid selected was hybrid.

2.3 Linear regression model and empirical Tixier equation

We have also compared the FT and NF models with two of the most commonly used approaches to calculate permeability from porosity, i.e., a linear regression

(LR) model and the Tixier equation. In the case of the LR model, a simple linear fit between the logarithms of the core values of permeability and porosity was performed:

$$\log(k) = a\log(\phi) + b \tag{9}$$

Tixier [3], using empirical relationships between water saturation, resistivity, and capillarity pressure, developed a method to obtain the permeability through the porosity (ϕ) and the irreducible water saturation (S_{wi}):

$$k^2 = 250\frac{\phi^3}{S_{wi}} \tag{10}$$

3 Results

The coefficients and the exponents of the general fractal permeability–porosity relationship (Eq. 1) were adjusted for the data of this study:

$$m=(2.01 \pm 0.01),\ c_1=(0.43 \pm 0.06),\ D=(2.33 \pm 0.01)$$

$$k = 596\phi + 16{,}180\phi^{2.02} + 1{,}185(10\phi)^{9.54} \tag{11}$$

In Fig. 2, the equation obtained in this study (i.e., Eq. 11) is compared with Eqs. 6 to 8. Equation 11 lies close to the equation that fits the data of the Rotliegend clean sandstone (FTRS, Eq. 7). Also, Eq. 11 lies above the equation proposed for the average type of sandstones (FTAS, Eq. 6) and far from and above that proposed for a shaly sandstone (FTSS, Eq. 8). The interval of the PX12 well studied here is characterized by the presence of massive sands (shallow zone) that turn to mainly clean thick sands interbedded with thin shale layers. The sand content is high through the entire section. This is in agreement with the results obtained, i.e., a good agreement with the Rotliegend clean sandstone equation, and values above and far from the shaly sandstone equation.

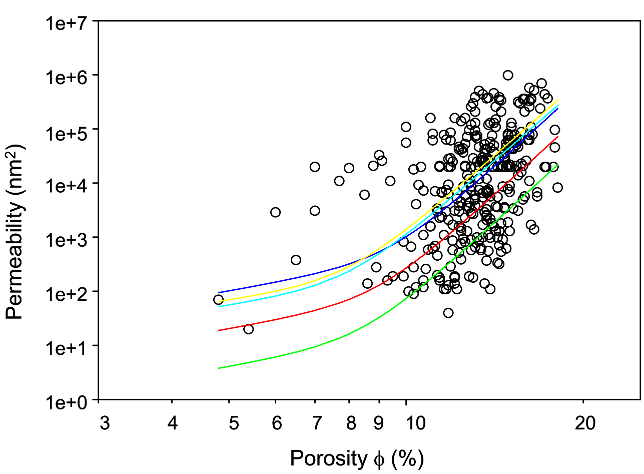

Fig. 2 Log–log plot of permeability versus porosity core data (*circles*) of well PX12. The fractal equation adjusted in this work using 100% (*yellow line*) of the core data is shown, together with the fractal equations derived and adjusted for other lithologies [1]: average type of sandstone (*red line*), Rotliegend sandstone (*dark blue line*) and shaly sandstones (*green line*). The equation obtained using 25% of the core data, randomly taken, is also included (*light blue line*)

For the NF model, after training the ANFIS, we have obtained the following NF equations:

R_i: If ϕ is Low
Then $\text{Log}(k) = 6.98\phi + 3.62$
R_i: If ϕ is Medium
Then $\text{Log}(k) = 20.05\phi + 0.26$
R_i: If ϕ is High
Then $\text{Log}(k) = 149.9\phi - 5.94$

The porosity coefficients in the equations are all positive. This behavior corresponds to an increase in the permeability with the porosity that is in agreement with the physical interpretation of the relationship between permeability and porosity given by the KC model [2].

The best adjustment was measured via the root mean-square error (RMSE):

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\log(y_i^{\text{core}})^2 - \log(y_i)^2)}{n}} \tag{12}$$

where n is the number of data points, y_i^{core} is the ith permeability core value, and y_i is the adjusted permeability. These values are presented in Table 1. In Fig. 3, we present the logs of calculated permeability for the LR, empirical Tixier (ET), and FTAS (Eq. 6) models. Figure 4 presents the results of the FT (particularized in this work, i.e., Eq. 11) and NF models using 100% of the core data. The 25% core data, randomly taken, and the results of the FT and NF obtained for these reduced data are also presented in Table 1 and Fig. 4.

The NF approach has better modeled the qualitative behavior of the permeability core data, especially in the shallow zone, as can be observed after comparing Figs. 3 and 4. In general, for all the models, the best qualitative prediction is obtained for the shallow zone. This zone is characterized by the presence of cleaner sands, and this could be the reason for the results obtained.

According to the RMSE values, the best fit is obtained using the NF approach. The FT (Eq. 11) and ET approaches have nearly the same RMSE values and do not improve the results obtained with the NF theory.

Table 1 The root mean-square error ($RMSE_I$) values for all the models (I= {LR, TE, FTAS, FT, and NF}) using 100% and 25% of the data, randomly taken

Data density	$RMSE_{LR}$	$RMSE_{TE}$	$RMSE_{FTAS}$	$RMSE_{FT}$	$RMSE_{NF}$
100%	1.10	0.95	1.02	0.94	0.86
25%	–	–	–	0.92	0.88

It is important to notice that, as we have compared different models to predict permeability from porosity in this work, the NF results were achieved using just one variable, i.e., porosity. Nevertheless, for the NF model, it is possible to use other variables (e.g., GR values) to increase the predictive fitness.

Regarding the FT results, the approach derived from Pape et al. [1] is intended for a wide porosity and permeability range. In fact, low, high, and medium porosity ranges are permissible in the equation, since, for a given range, the other two porosity ranges in the fractal equation do not contribute significantly due to the difference in powers of the porosity variable. It is important to notice that the studied data present a wide permeability range associated to a narrow porosity range. This distribution is not a usual one and is basically different from that corresponding to the data analyzed by Pape et al. [1]; nevertheless, a good approach is obtained with the fractal equation adjusted here. The FTAS also predicts the general behavior of the permeability data, as can be observed in Fig. 3, though a better RMSE value is obtained for the FTRS, as was explained above. This indicates that if the lithology of the studied well or column is known, it is possible to directly apply one of the particularized equations derived by Pape et al. for different lithologies. In this way, it is possible to predict the permeability without training the system, as is required with the NF approach, or without using additional information (e.g., the cementation value) in order to derive a particular equation for the study basin.

The clustering of the porosity core data in a narrow range could also be the reason for the relatively good

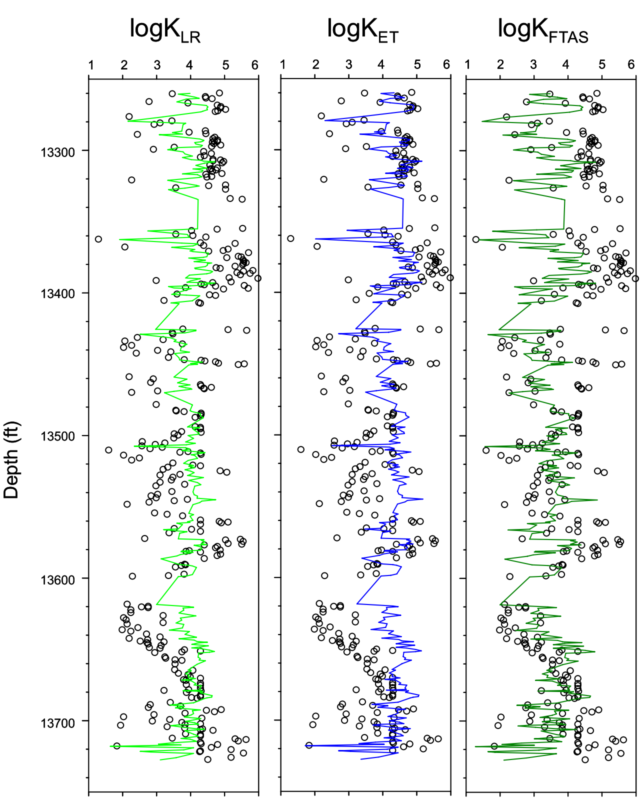

Fig. 3 Logs of calculated and predicted permeability for the linear regression (*green line*), Tixier equation (*blue line*), and average type of sandstone (*dark green line*) models. Core data (*circles*) are also included

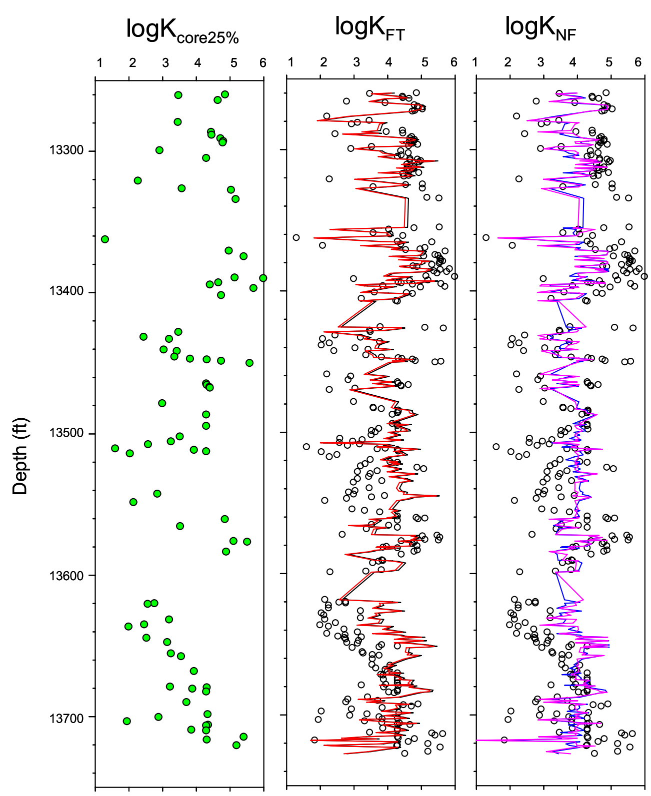

Fig. 4 Logs of calculated and predicted permeability obtained for the FT (Eq. 11) and NF models using 100% of the core data (*red* and *blue lines*, respectively) and 25% of the core data randomly taken (*black* and *pink lines*, respectively). The 25% core data randomly taken are presented in the *figure to the right* (*green circles*). Core data values are presented together with the logs of the models (*black circles*)

fitness obtained with the Tixier empirical relationship (see Fig. 3 and Table 1), that includes just one power, and, hence, mainly one range, for the porosity variable. Although the LR approach follows the general trend of the core permeability data (see Fig. 3), it tends to underestimate high permeability values, giving the worst fit. The reason for this could be that a regression model in general calculates mean values [2].

Regarding the data density, for the NF model, greater data densities (100% of the data as input) give the best prediction compared with lower densities (25% data), as was expected for a statistical approach (see Fig. 4 and Table 1). On the contrary, the fractal equation obtained with 25% of the data, randomly taken, gives a lower RMSE value compared with the equation adjusted with 100% of the data as input. The KC model was developed assuming ideal pore conditions [1]. In this sense, the ideal KC model could be reached with the decrease of the data density. This could explain the slightly lower RMSE value obtained when the fractal equation is derived with 25% of the data as input.

4 Conclusions

In this work, different approaches have been applied to predict permeability values from core data at well PX12, Maracaibo Lake: NF and fractal-based models, a LR model, and the Tixier empirical equation. The results obtained in this work indicate that, for the studied data, the best approach to permeability from porosity is obtained with the statistical approach based on the NF theory. The increase of the input data density in this case improves the results, as is expected for a statistical analysis. On the contrary, for the physical model based on the FT, the diminution in the data density could allow reaching the ideal theoretical KC model, improving the permeability prediction. The Tixier and LR approaches do not improve the results obtained with the NF or fractal-based models.

One of the advantages of the FT model over the NF model is that, if the lithology is known, one of the specific equations proposed by Pape et al. for different lithologies can be applied to obtain permeability data using as input just the porosity data. In this case, no additional parameters or the training of the system is required.

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